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# Terahertz radiation from a pipe with small corrugations ☆

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ABSTRACT

We have studied through analytical and numerical methods the use of a relativistic electron bunch to drive a metallic beam pipe with small corrugations for the purpose of generating terahertz radiation. For the case of a pipe with dimensions that do not change along its length, we have shown that—with reasonable parameters—one can generate a narrow-band radiation pulse with frequency $\sim 1$ THz, and total energy of a few milli-Joules. The pulse length tends to be on the order of tens of picoseconds. We have also shown that, if the pipe radius is tapered along its length, the generated pulse will end up with a frequency chirp; if the pulse is then made to pass through a compressor, its final length can be reduced to a few picoseconds and its peak power increased to $\sim 1$ GW. We have also shown that wall losses tend to be significant and need to be included in the structure design.



## 1. Introduction

For applications in fields as diverse as chemical and biological imaging, material science, telecommunication, semiconductor and superconductor research, there is great interest in having a source of short, intense pulses of terahertz radiation. There are laser-based sources of such radiation [1,2], capable of generating *e.g.* (several-cycle) pulses with frequency over the range 10–70 THz and energy of 20 μJ [3]. And there are beam-based sources, utilizing short, relativistic electron bunches [4,5]. One beam-based method impinges an electron bunch on a thin metallic foil and generates coherent transition radiation (CTR). Recent tests of this method at the Linac Coherent Light Source (LCLS) have obtained single-cycle pulses of radiation that is broad-band, centered on 10 THz, and contains $>0.1$ mJ of energy [6]. Another beam-based method generates THz radiation by passing a bunch through a metallic pipe with a dielectric layer [7]. At UCLA this method was used to generate narrow-band pulses with frequency 0.4 THz and energy 10 μJ. In this report we investigate a similar idea, that of using a short, relativistic beam to generate THz radiation in a metallic pipe with small corrugations, to explore what the possibilities of this approach might be.

It has been noted in the past, in the study of wall-roughness impedance, that a beam passing through a metallic pipe with small-scale corrugations excites a high-frequency mode that propagates with the beam. We propose introducing a short driving bunch into such structure, with aperture on the order of a millimeter and length on the order of a meter, in order to generate a pulse of radiation with frequency on the order of a terahertz and energy on the order of milli-Joules. With a pipe where the corrugations do not vary along the structure, we shall see that we obtain a narrow-band pulse that is relatively long, on the order of tens of picoseconds. To obtain a shorter pulse, one can imagine using a corrugated pipe that varies along its length, in order to introduce a frequency chirp in the pulse. If this is followed by a properly designed dispersive device—analogous to what is done in chirped pulse amplification (CPA) in high power lasers—the pulse can then be compressed, at the expense of being more broad-band than in the previous, unchirped case.

In this report we will study both ideas, THz pulse generation in a corrugated pipe that does not vary along its length and in one that does. In the second case, however, we will not attempt to solve the entire problem. We will focus on the first part of the process—generating the chirped pulse. Pulse generation in a corrugated pipe is studied using analytical formulas and numerical simulations. For the simulations we employ Zagorodnov's program ECHO, which computes the fields generated by an ultra-relativistic bunch in a structure in the time domain [8]. We begin with lossless beam pipe walls, and then include wall losses in the calculations. The pulse compression that follows is only treated conceptually, by simulating the effect of an ideal dispersive compressor. It is understood however, that working out an actual design for such a compressor will be a crucial component in getting a short, high peak power THz pulse from a pipe with small corrugations.

## 2. Theory

### 2.1. Analytical estimates

Consider a short, ultra-relativistic bunch of electrons passing on-axis through a periodic (cylindrically symmetric) metallic

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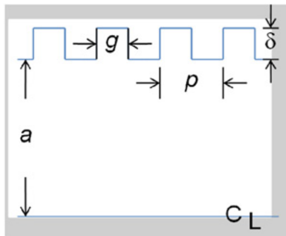


**Fig. 1.** A sketch of a part of the corrugated structure.

structure with small corrugations. Let the pipe radius be $a$, and the corrugations have depth $\delta$, period $p$, and gap $g$, with $\delta, p \ll a$ (see Fig. 1). When $\delta \gtrsim p$ (a case that we call "steeply corrugated") the beam excites one dominant mode at a frequency that is far above cut-off, in a pulse that follows the beam near the speed of light, $c$ (many higher frequency weak modes are also excited).[1] For the dominant, fundamental mode the wave number, $k$, and group velocity, $v_g$, are well approximated by [9,10]

$$k = \frac{2}{\sqrt{a\delta}}, \tag{1}$$

$$\left(1-\frac{v_g}{c}\right) = \frac{2\delta}{a} \ll 1. \tag{2}$$

Here, and in the rest of the report, for simplicity we have let the gap $g = p/2$ (for the general case, when this relation does not hold, the analytical formulas can be found in Refs. [9,10]).

A schematic of how the pulse is generated, and how it would arrive at a monitor located at the structure end is shown in Fig. 2. As the beam (the elliptical symbol at the bottom in frames a–c) traverses the structure, parts of a radiation pulse are continually being generated by it and then fall behind. Finally, at the field monitor the parts of the pulse arrive in the reverse order in which they were created. If the pipe length is $L$, then Eq. (2) implies that the length of the radiation pulse at the downstream end is

$$\ell = \frac{2\delta L}{a}. \tag{3}$$

Let us assume, for the moment, that the walls are perfectly conducting. Then the energy in the pulse at the end of the corrugated pipe is approximately equal to the energy lost by the exciting bunch during its passage through the pipe. And this loss, in turn, is approximately given by the loss to the fundamental mode alone (assuming a steeply corrugated structure, *i.e.* $\delta \gtrsim p$). (In the simulations, to be presented below, we will confirm these statements.) The energy loss (or "wake loss") is given by $U_w = Q^2\kappa L$, with $Q$ the charge in the bunch and $\kappa$ the loss factor [9–11]

$$\kappa = \frac{Z_0 c}{2\pi a^2} e^{-k^2\sigma_z^2}, \tag{4}$$

where $Z_0 = 377\ \Omega$ and $\sigma_z$ is the rms bunch length. Note that with a very short bunch ($k\sigma_z \ll 1$), $\kappa = Z_0 c/2\pi a^2$, and the wake loss is the largest (steady-state) loss in any structure with minimum aperture $a$ and length $L$ (see *e.g.* discussion in Ref. [12]). The power in the pulse is then given by $P = cU_w/\ell$.

As a practical example, consider a beam pipe with $a$=2 mm, $\delta = 50\ \mu$m, $p = 40\ \mu$m, $L$=50 cm, and an exciting bunch with $\sigma_z = 100\ \mu$m, $Q$=1 nC. For this example, the analytical formulas give: frequency $f = kc/2\pi = 0.3$ THz; $k\sigma_z = 0.63$, bunch energy loss $U_w$=1.5 mJ (for a point charge the loss would be $U_w$=2.25 mJ); power $P$=15 MW; and final pulse length $\ell = 2.5$ cm. We see that the frequency is on the order of 1 THz, the energy in the pulse is significant, and the pulse is quite long.

Some notes to consider concerning the results of this section:

1. All of our analytical results are valid for the case of a steeply corrugated structure ($\delta \gtrsim p$). If this condition is slightly violated, the mode frequency will be higher and the excitation (loss factor) lower than the analytical values. In the regime of a "shallowly corrugated" structure ($\delta \ll p$) the dominant mode is gone, and the structure has a completely different behavior.
2. The results given here are steady-state results. When a beam first enters a corrugated pipe there is a different, transient response that we have ignored. After a distance on the order of the catch-up distance, $z_{cu} = a^2/2\sigma_z$, the analytical formulas become valid. We will see that for the size of parameters discussed in this report, the transient distance is relatively small and transient effects can indeed be ignored.
3. The radiation pulse generated by the corrugated structure will be cylindrically symmetric of radius $a$. The fields vary linearly with radius: $E_r = H_\phi = H_0 r/a$, with $E_r$ the radial electric field, $H_\phi$ the azimuthal magnetic field, and $H_0$ a constant [13].
4. Resistive wall losses, which are ignored here, will significantly affect the pulse energy and peak power that can be achieved. This important issue will be addressed in a later section of this report.

## 2.2. Tapered structure

The pulse power can be enhanced if it is compressed to a fraction of a centimeter. This can be achieved by generating a terahertz pulse whose frequency varies from head to tail, and sending it through a dispersive system in which the head. We will call such a system a compressor. Mathematical representation of the compressor action of the pulse is discussed in the next subsection.

To introduce a frequency chirp into the pulse, we have considered adiabatically varying the corrugation parameters along the pipe. One of the simplest methods is to keep the actual corrugations unchanged, and to just vary the beam pipe radius gradually. Fig. 3 gives a sketch of the idea: the beam passes through a pipe that becomes gradually smaller. At the end there is a monitor to measure the pulse. This is followed by, at the moment, an ideal pulse compressor. In the configuration shown the front of the pulse will have a higher frequency than the back. However, the inverse configuration—small-to-large beampipe—should, in principle work as well.

The parameters for the tapered, corrugated structure and for the exciting bunch to be used in simulations presented below are given in Table 1. In particular, the pipe radius varies linearly from $a$=2 mm in the beginning to 1 mm at the end. The taper is gradual, so we assume the equation for $k$ (Eq. (1)) is valid locally; averaging along the structure, we estimate the central frequency to be 0.35 THz, and the bandwidth 0.125 THz. Note that estimating the final radiation pulse length is not so easy, and requires a detailed knowledge of the dispersion curve of a corrugated pipe—something that is beyond the scope of this report.

The analytical approximation of the energy in the radiation pulse, given by the wake energy, is $U_w = Q^2 L\langle\kappa\rangle$, with $\kappa$ given in Eq. (4) and the brackets means to average along the length of the pipe. For our linear taper the average can be performed explicitly, giving

$$U_w = \frac{Z_0 c}{8\pi}\frac{Q^2 L\delta}{\sigma_z^2(a_f-a_0)}\left[\exp\left(\frac{-4\sigma_z^2}{a_f\delta}\right)-\exp\left(\frac{-4\sigma_z^2}{a_0\delta}\right)\right], \tag{5}$$

with $a_0$ ($a_f$) the initial (final) pipe radius.

[1] Note that if the beam moves through the structure slightly off axis, a dominant dipole mode of the same frequency will also be excited.