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Beam matching strategies in undulators for SASE FEL operating devices

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ABSTRACT

We discuss the new problems emerging in charged beam transport for SASE FEL dynamics. The optimisation of the magnetic transport system for future devices requires new concepts associated with the slice emittance and the slice phase space distribution. We study the problem of electron beam slice matching and guiding in transport devices for SASE FEL emission by discussing matching criteria and how the associated design of the electron transport line may affect the FEL output performances. We analyse different matching strategies by studying the relevant effect on the FEL output characteristics.

1. Introduction

This paper is devoted to the problem of electron beam slice matching and guiding in transport devices for self amplified spontaneous emission (SASE) free electron laser (FEL) [1]. We will discuss matching criteria and how the associated design of the electron transport line may affect the FEL output performances.

The concept of slice emittance is a by-product of the SASE FEL Physics. It is indeed associated with the fact that, in these devices, the combination of mechanisms like gain, slippage and finite coherence length

$$l_c = \frac{\lambda}{4\pi\sqrt{3}\rho} \tag{1}$$

(with λ and ρ being the FEL operating wavelength and the Pierce parameter, respectively) determines a kind of local interaction, because the radiation experiences only a portion of the beam, having the dimensions of a coherence length (see Fig. 1). The interaction is therefore sensitive to the longitudinal and transverse characteristics of this "slice", which will be characterised by a specific six dimensional phase space distribution.

In Fig. 1(a) we have reported an example of a coherent seed, having an rms length of the order of the coherence length undergoing a high gain FEL amplification process, induced by an electron bunch with an rms length $\sigma_z \gg l_c$. In the case of SASE the laser field grows from the noise and therefore, when coherence develops, we have the formation of a number of peaks ($n \cong (\sigma_z/l_c)$) (identified with the Supermodes [2]). The evolution dynamics is particularly complex and the interplay between these modes during the growth is responsible for the characteristic spiking behaviour [3].

Within certain limits, we can consider the evolution of the field generated by an individual spike as due to the characteristics of the corresponding electron bunch slice. The growth of each spike will be therefore determined by various effects associated with the emittance, transverse section and matching condition, energy spread of the slice, determining the local interaction.

In Ref. [4] we started a preliminary analysis in this direction and studied the effect of the evolution of the Twiss parameters on the laser field evolution, in this paper we will discuss the problem more thoroughly and start with a more complete mathematical analysis, employing concepts from the geometry of conics and therefore we review first some geometrical properties of the ellipses, useful for the analysis of e-bunches phase space slicing.

We either use method of elementary analytical geometry and slightly more advanced techniques employing the formalism of quadratic forms.

We will initially consider the two-dimensional transverse phase space x,x' and define in it the Courant Snyder ellipse [2,5], centred at the origin of the axis (see Fig. 2) and specified by the following equation:

$$\gamma x^2 + 2\alpha x x' + \beta x'^2 = \varepsilon \tag{2a}$$

where ε is the beam emittance and the coefficients γ, α, β are the Twiss parameters, linked by the identity

$$\beta\gamma - \alpha^2 = 1 \tag{2b}$$

which ensures the normalisation of the phase space distribution

$$\Phi(x,x') = \frac{1}{2\pi\varepsilon}\exp\left[-\frac{1}{2\varepsilon}(\gamma x^2 + 2\alpha x x' + \beta x'^2)\right]. \tag{3}$$

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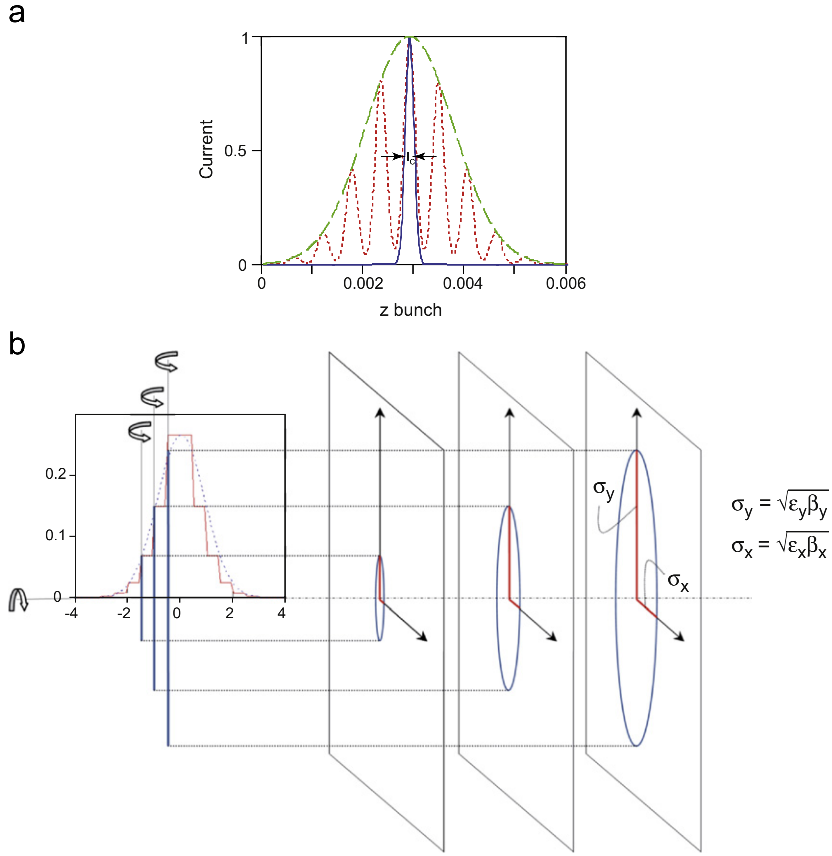

Fig. 1. (a) Local interaction and slice emittance, the dash line is the bunch envelope, the dot line represents the slices and the continuous line is the radiation distribution associated with the slice, l_c being the coherent length; (b) transverse sections of the sliced bunch.

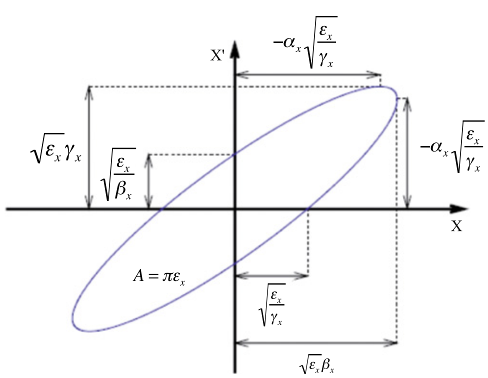

Fig. 2. The Courant Snyder ellipse and the relevant geometrical interpretation.

Furthermore the emittance and the Twiss coefficients [2,5] define the e-beam rms transverse length, divergence and correlation, according to the relations:

$$\sigma_x = \sqrt{\langle x^2 \rangle} = \sqrt{\beta\varepsilon}$$
$$\sigma_{x'} = \sqrt{\langle x'^2 \rangle} = \sqrt{\gamma\varepsilon}$$
$$\sigma_{x,x'} = \langle xx' \rangle = -\alpha\varepsilon \tag{4a}$$

where the average is taken on the distribution (3).

The geometrical interpretation of the various quantities, we have mentioned, is given in Fig. 2.

It is worth noting that the two vertical t and horizontal h tangents to the ellipse meet the conic at the points:

$$T_1 \equiv \left(\sqrt{\beta\varepsilon}, -\alpha\sqrt{\frac{\varepsilon}{\beta}}\right)$$

$$T_2 \equiv \left(-\alpha\sqrt{\frac{\varepsilon}{\gamma}}, \sqrt{\gamma\varepsilon}\right). \tag{4b}$$

The geometrical meaning of the correlation $\sigma_{x,x'}$ emerges, therefore, by an inspection of Fig. 2 and is interpreted as the area of the rectangle having as dimensions the coordinates of the tangent points.

The physical role of the α coefficient is that of quantifying the correlation between positions and momenta of the particles in the beam and a further geometrical role is played by its sign, specifying the orientation of the ellipse, which points in the positive direction of the axis for negative values of α and vice versa when it is positive. Ellipses with the same Twiss parameters, but with different emittances, will be considered similar.

The angle ϑ (see Fig. 3), formed by the ellipse major axis with the positive direction of the x-axis, can be determined by performing the axis rotation (see below for a correct understanding of the physical dimensions involved in the rotation process)

$$x = X\cos(\vartheta) + X'\sin(\vartheta)$$
$$x' = -X\sin(\vartheta) + X'\cos(\vartheta) \tag{5}$$

and by requiring that the cross terms in $X\,X'$ vanish. This procedure, which is essentially that of reducing the ellipse to the normal form by imposing that the rotated reference axes coincide with the ellipse axis, yields:

$$tg(2\vartheta) = -\frac{2\alpha}{\gamma - \beta}, \quad \gamma > \beta. \tag{6}$$

The above equation as it stands may not appear correct, the Twiss coefficients except α are indeed not dimensionless quantities, and we have $[\beta] = [L], [\gamma] = [L^{-1}]$. The rotation in Eq. (5), which mixes a length (x) and dimensionless quantity (x'), is not