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A study concerning the reduction of emittance via non-uniform dipoles under the $\beta_{\min}$ constraint

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ABSTRACT

Low emittance is a primary goal of electron storage ring based light sources. Optics with low emittance always require low beta functions in dipoles, which implies that strong quadrupoles are needed and that natural chromaticity arises. The work presented in this paper demonstrates that by introducing dipoles with a longitudinal dipolar field-variation, the minimum emittance can be lowered even if the minimum beta function is constrained.

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1. Introduction

High brilliance is sought in electron synchrotron light sources. The emittance, which is inversely proportional to the brilliance, is a fundamental parameter in storage rings, and many efforts have been made to lower the emittance in previous designs. However, in electron storage rings with constant-field dipoles via the balance between radiation damping and quantum excitation, there is an achievable theoretical minimum emittance [1].

For a theoretical minimum emittance (TME) lattice using uniform field dipoles, there is no constraint on the optics parameters at either end of the dipoles. The minimum emittance is

$$\varepsilon_{\mathrm{TME}} = C_q \gamma^2 \theta_b^3 / 12\sqrt{15} J_x \tag{1}$$

This result is achieved when $\alpha_x = \eta'_x = 0$, $\beta_x = L_b/2\sqrt{15}$ and $\eta_x = L_b\theta_b/24$ are satisfied at the center of the dipole [2].

For a double-bend achromat (DBA) lattice using uniform field dipoles, zero η_x and η'_x are required at the entrance of the dipole. In this case, the minimum emittance is

$$\varepsilon_{\mathrm{DBA}} = C_q \gamma^2 \theta_b^3 / 4\sqrt{15} J_x \tag{2}$$

This is achieved when $\beta_x = 6L_b/\sqrt{15}$ and $\alpha_x = \sqrt{15}$ hold at the entrance of the first dipole [2], where $C_q = 55\hbar/32\sqrt{3}mc = 3.84 \times 10^{-13}$ m for electrons, γ is the Lorentz factor of the beam energy, J_x is the horizontal damping partition number and θ_b is the bending angle of dipole. L_b is the length of dipole, β_x and α_x are the Courant–Snyder parameters and η_x and η'_x are the dispersion and its derivative, respectively.

To break the theoretical minimum emittance barrier for further emittance reduction, a non-uniform dipole was introduced by A. Wrulich in 1992, and was later elaborated analytically by Nagaoka [3,4] and numerically by Guo and Raubenheimer [5]. Recently, Wang [6] gave new, simple formulas for computing the minimum emittance. In general, these works focused mainly on theoretical analysis.

However, in practice, researchers often use portions of uniform dipoles instead of non-uniform dipoles. In this paper, we consider a simplified model of a non-uniform dipole that consists of some adjacent uniform rectangular dipoles and analyze the results of this model under the condition that the minimum beta function is constrained in the non-uniform dipole.

2. Basic ideas

To minimize the horizontal natural emittance ε_x with a non-uniform dipole made of some adjacent uniform rectangular dipoles, we shall start with the following formula [7]:

$$\varepsilon_x = C_q \frac{\gamma^2}{J_x} \frac{\oint (H/\rho^3) ds}{\oint (1/\rho^2) ds} = C_q \frac{\gamma^2}{J_x} \frac{I_5}{I_2} \tag{3}$$

where ρ is the bending radius at position s, I_2 and I_5 are the radiation integrals and H is the so-called H-function given by

$$H = \gamma_x \eta_x^2 + 2\alpha_x \eta_x \eta'_x + \beta_x \eta_x'^2 \tag{4}$$

We consider the transfer matrix of a generalized dipole magnet, neglecting the weak focusing term proportional to the

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square of the curvature in the bending magnet. The transfer matrix is

$$M_{\text{bend}} = \begin{pmatrix} 1 & s & \frac{s^2}{2\rho} \\ 0 & 1 & \frac{s}{\rho} \\ 0 & 0 & 1 \end{pmatrix} \tag{5}$$

Given initial optics parameters β_0, α_0, γ_0, η_0 and η_0' at the bending magnet entrance, the horizontal optics functions evolve as

$$\beta(s) = \beta_0 - 2\alpha_0 s + \frac{1+\alpha_0^2}{\beta_0} s^2 \tag{6a}$$

$$\alpha(s) = \alpha_0 - \frac{1+\alpha_0^2}{\beta_0} s \tag{6b}$$

$$\gamma(s) = \gamma_0 \tag{6c}$$

$$\eta(s) = \eta_0 + \eta_0' s + \frac{s^2}{2\rho} \tag{6d}$$

$$\eta'(s) = \eta_0' + \frac{s}{\rho} \tag{6e}$$

Our model is a simplified, analytically solvable model of a non-uniform dipole made of some adjacent, constant-field dipoles. The optics functions in the nth piece are

$$\beta_n(s) = \beta_{n-1,E} - 2\alpha_{n-1,E} s + \frac{1+\alpha_{n-1,E}^2}{\beta_{n-1,E}} s^2 \tag{7a}$$

$$\alpha_n(s) = \alpha_{n-1,E} - \frac{1+\alpha_{n-1,E}^2}{\beta_{n-1,E}} s \tag{7b}$$

$$\eta_n(s) = \eta_{n-1,E} + \eta_{n-1,E}' s + \frac{s^2}{2\rho_n} \tag{7c}$$

and

$$\eta_n'(s) = \eta_{n-1,E}' + \frac{s}{\rho_n} \tag{7d}$$

where the subscript "n" denotes the nth piece and "$n-1$, E" denotes the end of the n − 1th piece. Therefore, we obtain the expressions of integral I_5 and I_2 in Eq. (3) as follows:

$$I_5 = \sum_{i=1}^{n} \left(\int_0^{L_i} \frac{\gamma_0 \eta_i(s)^2 + 2\alpha_i(s)\eta_i(s)\eta_i'(s) + \beta_i(s)\eta_i'(s)^2}{\rho_i^3} ds \right) \tag{8a}$$

$$I_2 = \sum_{i=1}^{n} \frac{L_i}{\rho_i^2} \tag{8b}$$

In accordance with the standard constant field case, we shall express the minimum emittance in a familiar form

$$(\varepsilon_x)_{\min} = M C_q \frac{\gamma^2}{J_x} \theta_0^3 \tag{9}$$

The factor M for constant field dipole is given by

$$M_{\text{TME}} = \frac{1}{12\sqrt{15}}$$

and

$$M_{\text{DBA}} = \frac{1}{4\sqrt{15}} \tag{10}$$

The Appendix gives the results of the model in DBA and TME conditions when the beta function is not constrained. From these results, we can see that the minimum emittance can reach a very low value, but the minimum beta function becomes very low at the same time. In the lattice design, if we desire a small beta, there will be strong focusing quadrupoles that will increase the natural chromaticity and decrease the dynamic aperture. In this paper, we constrained the minimum beta as well as the one in the uniform dipole case when the minimum emittance was reached.

3. Minimum emittance with minimum beta function constraint

Keeping the bending angle constant and assuming the same minimum beta function in dipoles, we calculate the emittance reduction factor between a non-uniform dipole and a constant field dipole under the achromat condition and the TME condition, respectively. In our calculation, there are two non-uniform dipole models: one consists of constant dipoles with the same length but different radii, and the other has different lengths and different radii. We choose, for instance, a non-uniform dipole made of two uniform dipoles in the DBA condition and three in the TME condition that have the same length. The same process can be applied to the calculation of more pieces of uniform dipoles both with the same length and different lengths.

3.1. Minimum emittance under TME condition

The theoretical minimum emittance for a storage ring is obtained if both the horizontal beta and dispersion functions have a minimum value in the middle of the bending magnet. We consider a dipole made of pieces of uniform dipoles, the number of pieces is n (n is odd). When the storage ring reaches the theoretical minimum emittance with uniform dipoles, the minimum beta function is only in the middle of the dipole, $\beta_{\min} = L/2\sqrt{15}$.

Here, we take a dipole composed of three uniform dipoles for consideration. Let $\rho_1 = \rho_3 = \rho\mu$, $\rho_2 = \rho$ and $L_1 = L_2 = L_3 = L/3$, where L is the total length of the non-uniform dipole. Choosing the middle of the second dipole as the point $s=0$, here, the Twiss function $\beta_0 = \beta_{\min} = L/2\sqrt{15}$, $\alpha_0 = 0$, $\eta = 0$. The factor M will be

$$M = \frac{L^4 t_1 - 720\eta_0 L^2 \rho t_2 + 77760\eta_0^2 \rho^2 t_3}{96\sqrt{15}(2+\mu)^3(2+\mu^2)} \tag{11}$$

where $t_1 = 592 + \mu(344 + 102\,\mu + 15\,\mu^4)$, $t_2 = 20 + 6\mu + \mu^4$ and $t_3 = \mu^2(2+\mu^3)$.

The minimum emittance is obtained by imposing the condition that

$$\frac{\partial M}{\partial \eta_0} = 0 \tag{12}$$

We can then obtain the dispersion in the middle of the non-uniform when reaching the minimum emittance

$$\eta_0 = \frac{\mathrm{L}^2\,(20+6\mu+\mu^4)}{216(2\mu+\mu^4)\,\rho} \tag{13}$$

Substituting into Eq. (11), we can obtain

$$M/M_{\text{TME}} = \frac{194 + 108\mu + 54\mu^2 + 222\mu^3 + 104\mu^4 + 42\mu^5 + 5\mu^8}{3(2+\mu^3)(2+\mu^2)(2+\mu)^3} \tag{14}$$

Fig. 1 shows M/M_{TME} as a function of the parameter μ. The horizontal coordinate is μ and the vertical coordinate is M/M_{TME}. When $\mu \to 1$, the dipole becomes a constant field dipole, $M/M_{\text{TME}} \to 1$.

Table 1 gives the results of dipoles made of odd pieces of constant field dipoles with the same length.

In Table 1, the function "ρ_n" denotes the bending radius of the "nth" piece, and "$B_{\max}$" is the maximum field in the dipole composed of pieces of uniform dipoles. "B_0" is the field of a uniform dipole that has the same length and bending angle as the non-uniform dipole.

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