Nuclear Instruments and Methods in Physics Research A 654 (2011) 502–507

Contents lists available at ScienceDirect

Nuclear Instruments and Methods in Physics Research A

journal homepage: www.elsevier.com/locate/nima

# FEL beam characterization from measurements of the Wigner distribution function

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ARTICLE INFO

*Article history:*
Received 25 March 2011
Received in revised form
1 July 2011
Accepted 16 July 2011
Available online 23 July 2011

*Keywords:*
Wigner distribution
Spatial coherence
FEL beam

ABSTRACT

The Free-Electron-Laser FLASH at DESY has been characterized by a quantitative determination of the Wigner distribution function. The setup, comprising an ellipsodial mirror and a moveable extreme UV sensitive CCD detector, enables the mapping of two-dimensional phase spaces corresponding to the horizontal and vertical coordinate axes, respectively. For separable beams this yields the entire Wigner distribution, offering comprehensive information about spatial coherence properties, wavefront, beam profiles, as well as beam propagation parameters.

## 1. Introduction

A couple of FEL applications require a more complete beam characterization than can be gained from standard techniques as, e.g. caustic measurements [1], beam profiling [2] or wavefront sensing [3–6]. In particular for partially coherent sources the consideration of the 2nd order correlations, i.e. the mutual intensity [7] of the stochastic wave field, is mandatory for reliable beam propagation or for the design of complex optical systems. Regarding the emission of SASE-type FELs (SASE = self-amplified spontaneous emission), the stochastic nature of the beam generation and amplification process [8] leads to inherent instabilities and pulse-to-pulse fluctuations and, furthermore, influences the spatial coherence properties as predicted by theory [9,10] and supported by several experimental investigations [11–14]. The theoretical approach, however, usually makes some simplifying assumptions regarding system alignment, stability or, e.g. the electron beam shape, and uses extensive numerical simulations especially in the non-linear regime. The experiments, on the other hand, are based on Young's double slit experiment and investigation of the corresponding two-beam interference patterns. The latter, however, are rather difficult to evaluate and it is extremely time consuming to map the four-dimensional representation space of 2nd order correlation functions that way. In contrast, it is known [15–17] and has already been demonstrated for X-ray synchrotron sources [18,19], that the Wigner distribution [20], being a two-dimensional Fourier transform of the mutual intensity, can be fully reconstructed from two-dimensional intensity profiles of the beam via tomography, using refractive and/or reflective optics exclusively. A corresponding fully automated setup for the purpose of characterization of excimer laser beams has been described recently [21].

In this paper, after a brief review of the Wigner distribution approach, we present results of its reconstruction from caustic measurements at FLASH beam line 2 (BL2) at a mean wavelength of 10.8 nm. The evaluation of beam parameters including global degree of coherence, wavefront and modal composition of the FEL beam is described.

## 2. Theory

The Wigner distribution $h$ of a quasi-monochromatic paraxial beam is defined in terms of the mutual intensity $J$ as a two-dimensional Fourier transform of the latter according to [22]:

$$h(\mathbf{x},\mathbf{u}) = \left(\frac{k}{2\pi}\right)^2 \iint J(\mathbf{x}-\mathbf{s}/2,\mathbf{x}+\mathbf{s}/2)e^{ik\mathbf{u}\cdot\mathbf{s}}d^2s, \quad (1)$$

where $\mathbf{x}=(x, y)$, $\mathbf{s}=(s_x, s_y)$ are two-dimensional spatial and $\mathbf{u}=(u, v)$ angular coordinates in a plane perpendicular to the direction of beam propagation and $k$ the mean wave number of light, respectively. As $J$ is Hermitian, $h$ is real, although it may become negative in some regions. However, its marginal distributions with respect to $\mathbf{x}$ and $\mathbf{u}$ are always non-negative and yield the

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doi:10.1016/j.nima.2011.07.031

irradiance (near field) $I(\mathbf{x})$ and the radiant intensity (far field) $I_{FF}(\mathbf{u})$ respectively [20].

The propagation of the Wigner distribution through static and lossless paraxial systems, signified by a $4\times 4$ optical ray propagation ABCD matrix $S$ from an input ($i$) to an output ($o$) plane writes [15,20]:

$$h_i(D\mathbf{x}-B\mathbf{u},-C\mathbf{x}+A\mathbf{u})=h_o(\mathbf{x},\mathbf{u}). \tag{2}$$

Likewise, the four dimensional Fourier transform $\tilde{h}$ of $h$ obeys a similar transformation law under propagation [15]:

$$\tilde{h}_i(A^T\mathbf{w}+C^T\mathbf{t},\ B^T\mathbf{w}+D^T\mathbf{t})=\tilde{h}_o(\mathbf{w},\mathbf{t}), \tag{3}$$

where $(\mathbf{w},\mathbf{t})$ are the Fourier-space coordinates corresponding to $(\mathbf{x},\mathbf{u})$.

Considering a set $\{p\}$ of parameters and a set of irradiance profiles $I_{\{p\}}(x,y)$ recorded at positions which are connected to an arbitrary reference plane via the corresponding ray transformation matrices $S_{\{p\}}$, one obtains, according to the marginal property of $h$ and well known Fourier relations:

$$\iint h_{\{p\}}(x,y,u,v)d^2uv=I_{\{p\}}(x,y)\overset{FT}{\leftrightarrow}\tilde{I}_{\{p\}}(w_x,w_y)=\tilde{h}_{\{p\}}(w_x,w_y,0,0) \tag{4}$$

and from (3) and (4) [19]:

$$\tilde{h}_{ref}(A^T_{\{p\}}\mathbf{w},B^T_{\{p\}}\mathbf{w})=\tilde{I}_{\{p\}}(\mathbf{w}). \tag{5}$$

In particular, for a separable beam, the Wigner distribution can be written as a product

$$h(x,y,u,v)=h_x(x,u)\cdot h_y(y,v) \tag{6}$$

and, provided $S_{\{p\}}$ is of the aligned astigmatic type, i.e.,

$$S=\begin{pmatrix} A_x & 0 & B_x & 0\\ 0 & A_y & 0 & B_y\\ C_x & 0 & D_x & 0\\ 0 & C_y & 0 & D_y \end{pmatrix}$$

Eq. (4) and (5) decompose likewise leading to:

$$\tilde{h}_{ref,x}(A_{\{p\},x}w_x,B_{\{p\},x}w_x)=\tilde{I}_{\{p\},x}(w_x)$$
$$\tilde{h}_{ref,y}(A_{\{p\},y}w_y,B_{\{p\},y}w_y)=\tilde{I}_{\{p\},y}(w_y), \tag{5′}$$

$\tilde{I}_{x,y}$ being the Fourier-transformed marginal of $I(x,\,y)$. For the experimental arrangement described below, parameter $p$ corresponds to the detector position $z$ and, with $A_x=A_y=1$, $B_x=B_y=z$, Eq. (5′) writes:

$$\tilde{h}_{ref,x}(w_x,z\cdot w_x)=\tilde{I}_{\{z\},x}(w_x)$$
$$\tilde{h}_{ref,y}(w_y,z\cdot w_y)=\tilde{I}_{\{z\},y}(w_y). \tag{5″}$$

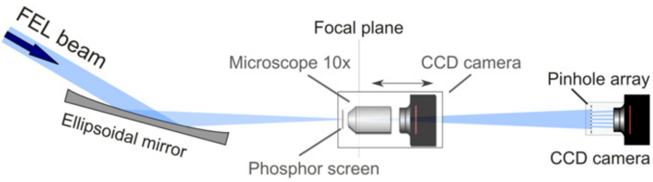

**Fig. 1.** Experimental setup for measurement of the separable Wigner distribution of the FLASH beam at BL2 (cf. text).

## 3. Experimental

Fig. 1 shows the setup employed for beam caustic measurements at FLASH BL2 at a wavelength of 25.9 nm. This experiment is described in detail [6] elsewhere; therefore, only the features relevant for evaluation of the Wigner distribution are briefly reported. A carbon-coated ellipsoidal mirror with 2 m focal length focuses the FEL beam to approximately 20 μm FWHM. Optimum alignment of the mirror was achieved by minimizing its wavefront aberration, as detected with a EUV Hartmann sensorinstalled in the FEL beam ~4820 mm behind the ellipsoidal mirror. Approximately 2.6 nm rms wavefront aberration was observed for best focusing conditions [6]. A niobium filter with 202 nm thickness was used to attenuate the beam both for the Hartmann and the caustic measurements. Due to the chromaticity of this filter, the fundamental at $\lambda=25.9$ nm is efficiently blocked and only the second and third harmonic at $\lambda=13$ and 8.6 nm are transmitted (pulse energy fractions of 0.35% for the second and 0.4% for the third harmonic [25]), leading to a mean wavelength of $\lambda=10.8$ nm for evaluation. The caustic sensor, a phosphorous screen imaged onto a CCD chip by a 10× magnifying microscope on a linear translation stage, senses the FEL beam near the focal plane of the ellipsoidal mirror. Beam profile measurements were taken at $N_z=32$ $z$-positions within an interval of 124 mm evenly distributed around the beam waist and covering approximately 2.5–3 Rayleigh lengths ($z_R$) in both axial directions. At each $z$-position 100 single pulses were recorded.

Data evaluation starts with background correction and determination of the centroids and 2nd order spatial moments $<x^2>$, $<y^2>$ and $<xy>$ for each profile, followed by a rotation of the reference frame in order to minimize the normalized mixed moments averaged over all measurement positions

$$\sigma=N_z^{-1}\sum\nolimits_i 2|\langle xy\rangle_i|/(\langle x^2\rangle_i+\langle y^2\rangle_i). \tag{7}$$

Provided the mixed moments $<xy>$ can be neglected after this rotation, a standard evaluation yields the 2nd order beam matrix [1] at the reference plane, from which important beam parameters as $M^2$, beam divergence and waist can be calculated [1] for $x$- and $y$-directions, respectively (zero twist has to be assumed).

For reconstruction of the Wigner function each rotated two-dimensional record is separately integrated over $x$ and $y$, delivering the one-dimensional marginal distributions $I_{x,y}$. Their Fourier transforms $\tilde{I}_{x,y}$ are then mapped into two-dimensional sub-spaces according to (5″), using the nearest neighbor method for interpolation onto a regular two-dimensional grid of size 256×256.

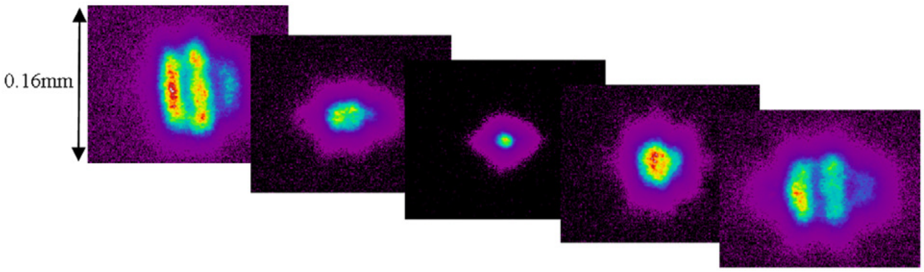

**Fig. 2.** FEL irradiance profiles at five positions in the vicinity of the beam waist. The measurement positions with respect to the camera reference frame are (from left to right) $z=0$ mm, $z=30$ mm, $z=60$ mm, $z=90$ mm and $z=120$ mm.