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Treatment of photon radiation in kinematic fits at future e^+e^- colliders

M. Beckmann [a], B. List [b,*], J. List [a]

[a] Deutsches Elektronen-Synchrotron DESY, Notkestr. 85, 22607 Hamburg, Germany
[b] University of Hamburg, Institute for Experimental Physics, Luruper Chaussee 149, 22761 Hamburg, Germany

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ABSTRACT

Kinematic fitting, where constraints such as energy and momentum conservation are imposed on measured four-vectors of jets and leptons, is an important tool to improve the resolution in high-energy physics experiments. At future e^+e^- colliders, photon radiation parallel to the beam carrying away large amounts of energy and momentum will become a challenge for kinematic fitting. A photon with longitudinal momentum $p_{z,\gamma}(\eta)$ is introduced, which is parametrized such that η follows a normal distribution. In the fit, η is treated as having a measured value of zero, which corresponds to $p_{z,\gamma}=0$. As a result, fits with constraints on energy and momentum conservation converge well even in the presence of a highly energetic photon, while the resolution of fits without such a photon is retained. A fully simulated and reconstructed $e^+e^-\rightarrow q\bar{q}q\bar{q}$ event sample at $\sqrt{s}=500$ GeV is used to investigate the performance of this method under realistic conditions, as expected at the International Linear Collider.

1. Introduction

Radiation of photons at angles so small that they escape along the beam pipe is usually not taken into account in kinematic fits. At previous e^+e^- colliders such as LEP, the losses due to photon radiation were acceptable [1]. At future facilities such as the International Linear Collider (ILC) [2] or the Compact Linear Collider (CLIC) [3], photon radiation will be much stronger due to higher centre-of-mass energies and stronger focussing of the beams, which makes it desirable to model photon radiation in kinematic fits.

Kinematic fitting is a well-established tool to improve jet energy and invariant mass resolutions. A number of four-vectors representing the final state particles is fitted under constraints such as energy and momentum conservation. The four-vectors are parametrized by suitably chosen variables such that the measured values follow an approximately Gaussian distribution around the true values. A χ^2 that quantifies the deviation between measured and fitted parameters is minimized under the condition that the imposed constraints are fulfilled [4].

The improvement in resolution emerges from the redundant information contained in the measured values in the presence of constraints. Unmeasured parameters reduce the redundancy, since one constraint is used up for each unmeasured parameter to determine its value. The redundancy is quantified by the number of degrees of freedom, which is given by the number of constraints minus the number of unmeasured parameters.

The two main effects that cause the emission of photons collinear with the incoming beams so that they escape the main detector are initial state radiation (ISR) and beamstrahlung. ISR is a higher-order QED effect, at which real photons are emitted before the actual interaction. Beamstrahlung is caused by the electrical fields of the bunches colliding with each other: electrons in one bunch are deflected by the field of the other bunch and thus emit bremsstrahlung photons.

ISR is characterized by an energy spectrum that follows a power law with an exponent of roughly -0.9 [5]. Thus the vast majority of events have at most one ISR photon with an energy above a few GeV, which is the accuracy to which the total energy and longitudinal momentum of fully hadronic events can be measured by a typical detector envisioned for the linear collider. This photon can, however, carry substantial energy of tens of GeV. Beamstrahlung on the other hand has an energy spectrum with an exponent of $-2/3$, but with an additional exponential suppression of high energy photons [6]. The mean number of beamstrahlung photons emitted prior to the interaction can be of order one or even larger, depending on the beam parameters.

This paper presents a novel method to take the energy and longitudinal momentum of photon radiation into account in kinematic fits. A priori information about the momentum spectrum of photon radiation is used to treat the photon's momentum as a measured parameter in the fit. As a test case, the production of W^+W^-/Z^0Z^0 pairs decaying to light quark jets at the ILC is considered, with fully simulated Monte Carlo events as

* Corresponding author.
E-mail address: Benno.List@desy.de (B. List).

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reconstructed by the International Large Detector (ILD) [7]. A more detailed description of the method and its application tests can be found in Ref. [8].

The main focus of this method is an improved treatment of the effects of ISR, because ISR is the main source of highly energetic photons. Therefore, only a single photon is included in the kinematic fit, with an energy spectrum given by a power law, as expected for ISR. A similar method with the inclusion of two photons in the fit and an energy spectrum describing the combined effects of ISR and beamstrahlung is the subject of ongoing work and beyond the scope of the current publication. However, the method presented here leads to a significant improvement also in the presence of beamstrahlung, as shown in Section 3.

2. Representation of the photon

Since photons from ISR and beamstrahlung escaping the detector have to a good approximation zero transverse momentum with respect to the beam direction, they affect mainly the conservation of (detected) energy E and longitudinal momentum p_z. The simplest method to cope with highly energetic photons in a constrained kinematic fit is therefore to drop the energy and longitudinal momentum conservation constraints, thus losing two degrees of freedom.

A somewhat better solution is to introduce a fit object representing the undetected photon with one free, unmeasured parameter, namely its longitudinal momentum $p_{z,\gamma}$, and set $p_{x,\gamma}=p_{y,\gamma}=0$ and thus $E_\gamma=|p_{z,\gamma}|$. This allows the energy and p_z constraints to be recovered, at the price of one unmeasured parameter, so that one degree of freedom is regained.

However, this approach neglects the information about the momentum spectrum of the photons. Here this information is used so that the photon is treated as a particle with a measured momentum of zero and an uncertainty derived from its known momentum spectrum.

2.1. Parametrization of the photon energy

In a kinematic χ^2 fit the measured four-vector components of a particle or jet are parametrized with parameters η_i (e.g., E,θ,ϕ) such that the difference $\eta_{\mathrm{i,\ meas}}-\eta_{\mathrm{i,\ true}}$ between the measured $\eta_{\mathrm{i,\ meas}}$ and the true value $\eta_{\mathrm{i,\ true}}$ follows a Gaussian distribution with zero mean and standard deviation $\delta\eta_{\mathrm{i}}$ (for reasons of notational simplicity we limit the discussion to the case where the parameters η_{i} are uncorrelated). Then,

$$\chi^2=\sum_i \frac{(\eta_{\mathrm{i,\ meas}}-\eta_{\mathrm{i}})^2}{\delta\eta_{\mathrm{i}}^2} \tag{1}$$

is, apart from a constant, proportional to the negative logarithm of the likelihood to obtain the measured values, given the values η_{i}:

$$\chi^2=-2\ln\mathcal{P}(\eta_{\mathrm{i,\ meas}}|\eta_{\mathrm{i}})+const. \tag{2}$$

Thus, the χ^2 fit seeks the best estimate η_{i} for the true parameter values by maximizing the likelihood to get the observed parameter values η_{meas} under the condition that the imposed constraints are fulfilled, which are expressed by a number of constraint functions $g_{\mathrm{k}}(\eta_{\mathrm{i}})=0$. No assumption is made, or is necessary, about the distribution of the true parameter values $\eta_{\mathrm{i,\ true}}$.

However, if an ensemble of events is considered where the distribution of a parameter η_{true} is known to be Gaussian with zero mean, then for this ensemble the choice $\eta_{\mathrm{meas}}=0$ also leads to a Gaussian distribution of $\eta_{\mathrm{meas}}-\eta_{\mathrm{true}}$, and for such an ensemble it appears justified to estimate η_{true} by means of a χ^2 fit.

In the case of photon radiation, the distribution of the unmeasured momentum $p_{z,\gamma}$ is known, though definitely non-Gaussian. Thus we seek a parametrization of the photon's momentum $p_{z,\gamma}=p_{z,\gamma}(\eta)$ such that the true value of η follows a Gaussian distribution with mean zero and unit standard deviation $\delta\eta=1$. Then the photon will be treated as if it had a measured value of $\eta_{\mathrm{meas}}=0$. The photon will then be added to the list of fit objects in the kinematic fit, thereby introducing an additional contribution to the overall χ^2 of $\eta^2/\delta\eta^2=\eta^2$. By this procedure, the a priori knowledge of the photon's energy spectrum (in particular the fact that it is negligibly small in most cases) is used, and all energy and momentum constraints can be applied.

The probability density function $\mathcal{P}(y)$ for the energy fraction $y=E_\gamma/E_{\mathrm{beam}}$ carried by initial state radiation is well approximated by [5]

$$\mathcal{P}(y)=\beta y^{\beta-1} \tag{3}$$

with the exponent β given by

$$\beta=\frac{2\alpha}{\pi}\left(\ln\frac{s}{m_{\mathrm{e}}^2}-1\right) \tag{4}$$

which corresponds to $\beta=0.1235$ for $\sqrt{s}=500\,\mathrm{GeV}$.

Considering that an ISR photon can be emitted by either beam leads to

$$\mathcal{P}(p_{z,\gamma})=\frac{\beta}{2E_{\max}}\cdot\left|\frac{p_{z,\gamma}}{E_{\max}}\right|^{\beta-1} \tag{5}$$

where $E_{\max}\le E_{\mathrm{beam}}$ is the maximum possible photon energy. As a consequence, the quantity z given by

$$z=\mathrm{sign}(p_{z,\gamma})\left(\frac{|p_{z,\gamma}|}{E_{\max}}\right)^{\beta} \tag{6}$$

is uniformly distributed between -1 and 1, and hence

$$\eta=\sqrt{2}\cdot\mathrm{erf}^{-1}(z) \tag{7}$$

follows a Gaussian distribution with zero mean and unit standard deviation. Here, $\mathrm{erf}^{-1}(z)$ denotes the inverse of the error function given by $\mathrm{erf}(x)=2/\sqrt{\pi}\int_0^x \mathrm{e}^{-t^2}\,\mathrm{d}t$.

Conversely the expressions for z and $p_{z,\gamma}$ as a function of the parameter η read

$$z(\eta)=\mathrm{erf}(\eta/\sqrt{2}) \tag{8}$$

$$p_{z,\gamma}(\eta)=\mathrm{sign}(z)E_{\max}|z|^{1/\beta} \tag{9}$$

$$=\mathrm{sign}(\eta)E_{\max}\left[\mathrm{erf}(|\eta|/\sqrt{2})\right]^{1/\beta}. \tag{10}$$

2.2. Properties of the parametrization

Fig. 1 shows a graph of $p_{z,\gamma}(\eta)$ for $E_{\max}=225\,\mathrm{GeV}$ and $\beta=0.1235$. The function has four distinct bends around $|\eta|\approx 1$ and ≈ 2.5. It is flat around $\eta=0$, reflecting the fact that the majority of ISR photons have negligible momentum; only for $|\eta|>0.7$ significant momenta above 1 GeV are predicted.

Around $\eta=0$ the value of $p_{z,\gamma}$ does not change and thus cannot influence the global χ^2 of the kinematic fit. Therefore, the penalty term η^2 leads to a local minimum of the χ^2 at this value of η. This will also be the global minimum if the measured four-momenta of the final state particles are compatible with no missing momentum from ISR. In this case, a fit with a photon fit object has exactly the same result as a fit without a photon.

Due to this local minimum, any minimization method based on derivatives will always yield $\eta=0$ if this value is used as starting value in the minimization. Therefore, to find the global