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Validation of analytical formula for the efficiency calibration of gamma detectors using coaxial and off-axis extended sources

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ABSTRACT

In this paper, we introduce a direct analytical mathematical method for calculating the geometrical and absolute total efficiencies of gamma NaI scintillation detectors using an arbitrarily positioned point source. Generalizing, the geometrical and absolute total efficiencies are extended to include coaxial and off-axis sources with plane and volumetric shapes. The efficiencies are deduced into straightforward mathematical expressions. The current theoretical and the published efficiency values are in good agreement.

1. Introduction

Gamma spectrometry is one of the tools commonly used for the measurement of various environmental radionuclides. Simultaneous determination of the absolute activity of gamma emitting samples in a wide energy range can be obtained. To calculate the absolute activity, the sample to detector absolute efficiency is needed. The calculation of sample to detector absolute efficiency using experimental, empirical and Monte Carlo approaches has been treated by several authors [1–7]. In this paper, by the use of an analytical method based on the direct mathematical method reported by Selim and Abbas [8–21], direct analytical formulae have been derived to calculate the geometrical and total efficiencies of cylindrical NaI scintillation detector for extended coaxial and off-axis circular disc and cylindrical sources.

2. Mathematical viewpoint

2.1. Point source

The absolute efficiency, for an arbitrarily positioned radiating point source placed at any distance from the detector can be represented by the following equation:

$$\varepsilon_{point} = \frac{1}{4\pi}\int_{\Omega}(1-\exp(-\mu d))\,d\Omega \tag{1}$$

where μ is the attenuation coefficient of the detector material without the coherent part [22], d is the travelled distance in the detector, and the solid angle subtended by a detector at an isotropic radiating point source is defined as:

$$d\Omega = \sin\theta\, d\varphi\, d\theta \tag{2}$$

where θ and φ are the polar and the azimuthal angles, respectively.

For the symmetry around the azimuth angle φ

$$\varepsilon_{point} = \frac{1}{4\pi}\int_{\theta}\int_{\varphi} f_i d\varphi d\theta \tag{3}$$

where, $f_i = (1-e^{-\mu.di})\sin\theta$.

The absolute efficiency, for an arbitrarily positioned radiating point source placed at a lateral distance ρ less than the detector radius R, Selim and Abbas [8] derived the following expressions:

$$\varepsilon_{point} = \frac{1}{2\pi}\left[\pi\int_{0}^{\theta_1} f_1\, d\theta + \int_{\theta_1}^{\theta_2}\int_{0}^{\pi} f_2\, d\varphi\, d\theta + \int_{\theta_1}^{\theta_3}\left(\varphi'_{\max} f_1 - \int_{0}^{\varphi'_{\max}} f_2 d\varphi\right) d\theta + \int_{\theta_2}^{\theta_4}\int_{0}^{\varphi_{\max}} f_2 d\varphi\, d\theta\right] \tag{4}$$

where, the travelled distances (d_i) inside the detector are:

$$d_1 = \frac{L}{\cos\theta} \tag{5}$$

$$d_2 = \frac{\rho\cos\varphi + \sqrt{R^2-\rho^2\sin^2\varphi}}{\sin\theta} - \frac{h}{\cos\theta} \tag{6}$$

The azimuth angles are:

$$\varphi_{\max} = \cos^{-1}\left(\frac{\rho^2 - R^2 + h^2\tan^2\theta}{2\rho h\tan\theta}\right) \tag{7}$$

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$$\varphi'_{\max} = \cos^{-1}\left(\frac{\rho^2 - R^2 + (h+L)^2 \tan^2\theta}{2\rho(h+L)\tan\theta}\right) \quad (8)$$

The polar angles are:

$$\theta_1 = \tan^{-1}\left(\frac{|R-\rho|}{h+L}\right) \quad (9)$$

$$\theta_2 = \tan^{-1}\left(\frac{|R-\rho|}{h}\right) \quad (10)$$

$$\theta_3 = \tan^{-1}\left(\frac{R+\rho}{h+L}\right) \quad (11)$$

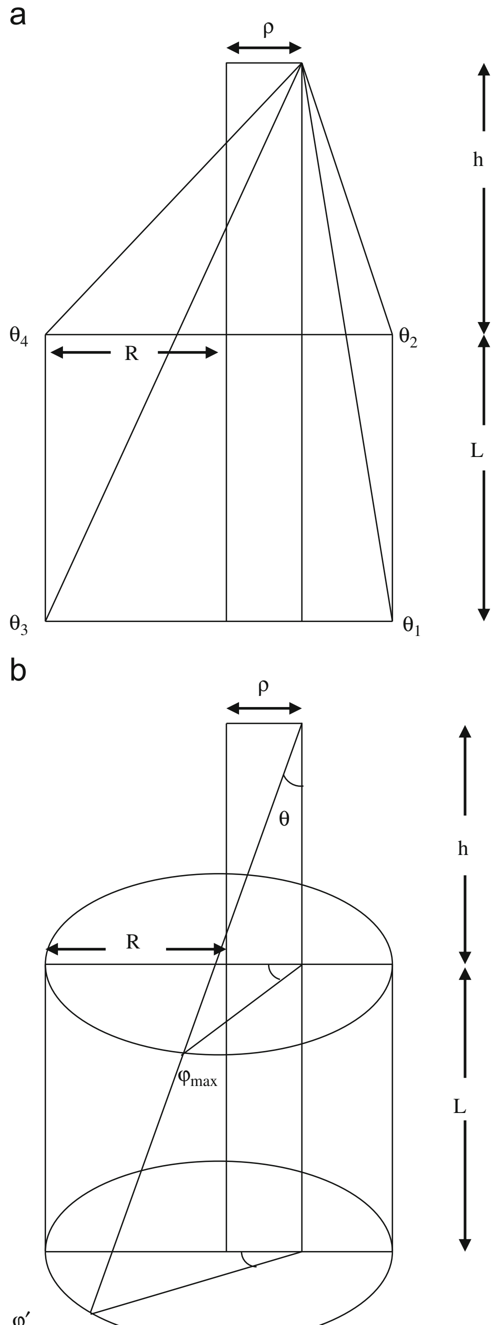

Fig. 1. A point source configuration at $\rho \le R$.

$$\theta_4 = \tan^{-1}\left(\frac{R+\rho}{h}\right) \quad (12)$$

The geometrical notations of R, L, h, and ρ are as shown in Fig. 1. Setting $\rho = 0$ in Eqs. (4–12), results in the same efficiency expressions for an axial-point source as derived by Irfan and Prasad [23]:

$$\varepsilon_{point} = \frac{1}{2}\left(\int_0^{\theta_1} f_1\, d\theta + \int_{\theta_1}^{\theta_2} f_2\, d\theta\right) \quad (13)$$

In the case of a point source located at a lateral distance $\boldsymbol{\rho}$ greater than the detector radius $\boldsymbol{R}$, as shown in Fig. 2, the absolute efficiency has been derived by Hamzawy [24]:

a) Case one ($\theta_3 \geq \theta_2$):

$$\varepsilon_{point} = \frac{1}{2\pi}\left[\int_{\theta_1}^{\theta_2}\int_0^{\varphi'_{\max}} f_3\, d\varphi\, d\theta + \int_{\theta_2}^{\theta_T}\left(\int_0^{\varphi_{\max}} f_1\, d\phi + \int_{\varphi_{\max}}^{\varphi'_{\max}} f_3\, d\varphi\right) d\theta + \int_{\theta'_c}^{\theta_T}\int_{\varphi'_{\max}}^{\varphi_c} f_4\, d\varphi\, d\theta + Y_1\right] \quad (14)$$

b) Case two ($\theta_3 < \theta_2$):

$$\varepsilon_{point} = \frac{1}{2\pi}\left[\int_{\theta_1}^{\theta_3}\int_0^{\varphi'_{max}} f_3\, d\varphi\, d\theta + \int_{\theta'_c}^{\theta_3}\int_{\varphi'_{max}}^{\varphi_c} f_4\, d\varphi\, d\theta + \int_{\theta_3}^{\theta_2}\int_0^{\varphi_c} f_4\, d\varphi\, d\theta + Y_2\right] \quad (16)$$

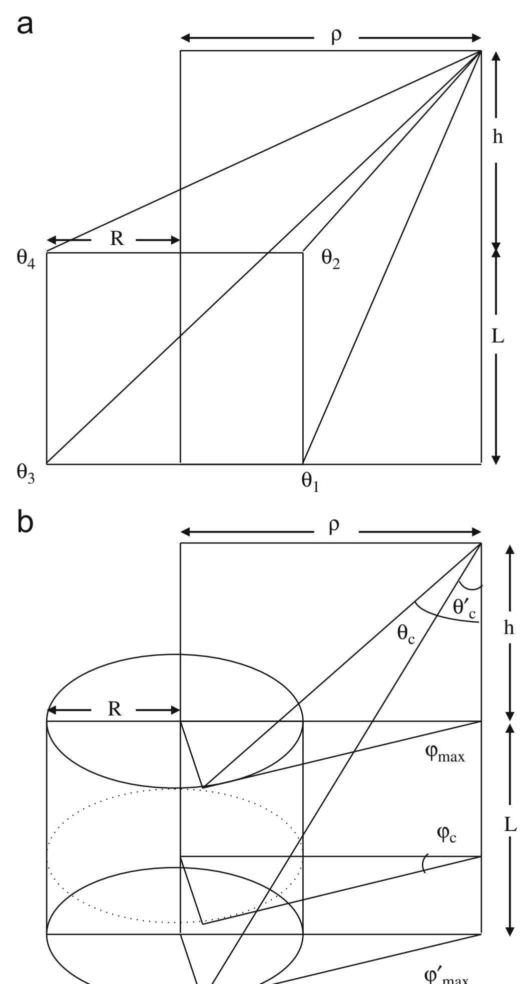

Fig. 2. A point source configuration at $\rho > R$.