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Entanglement between degrees of freedom of single neutrons

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ABSTRACT

Non-local correlations between subsystems sufficiently separated in spacetime have been extensively discussed in the light of the Einstein, Podolsky and Rosen (EPR) paradox, together with Bell's inequality. Within quantum terminology, such a non-locality can be interpreted as a consequence of the entanglement of subsystems. A more general concept, i.e., quantum contextuality, compared to non-locality, can be introduced to describe other striking phenomena predicted by quantum theory. As examples of quantum contextuality, we report several neutron interferometer experiments: a violation of a Bell-like inequality, a Kochen–Specker-like phenomenon, a quantum state tomography. Entanglement is achieved not between the particles, but between the degrees of freedom of a single-particle. Furthermore, an experiment dealing with triple entanglement is presented.

1. Introduction

As quantum mechanics predicts, particles – like neutrons – exhibit wave properties. A neutron interferometer, made of silicon perfect-crystal with a monolithic structure, was invented in the seventies [1] and has been used for many demonstrations of interference phenomena with neutrons. There, coherent beams are macroscopically separated (by typically several cm) which allows to insert macroscopic optical elements like a phase shifter or spin flippers into the beams. The first demonstration of the perfect crystal neutron interferometer was accomplished at the TRIGA reactor in Vienna and it was successfully operated at the ILL high flux reactor afterwards. Starting from a demonstration of the 4π-symmetry of the spinor [2,3], many fundamental experiments concerning striking quantum mechanical phenomena have been carried out [4].

Bell's inequalities together with the EPR-paradox arose considerable interest for many decades [5,6]. Tests of a local assumption are the main issue and how to explain quantum non-local correlations is still under discussion. There, entanglement, i.e., non-separable subspaces, forms the basis of striking features of quantum mechanics. Extending the local assumption to commuting observables, the more general concept of non-contextuality is introduced. Non-contextuality implies that the result of a measurement is independent of the previous or simultaneous measurement of any set of mutually commuting observables [7]. This is a simple classical deduction of independent measurements for the quantum phenomena. Neutron optical experiments with interferometers and polarimeters are suitable for the investigations of quantum contextuality due to their efficient and uncomplicated handling of beams.

We have accomplished polarized neutron interferometric experiments to demonstrate remarkable quantum contextual phenomena with entanglement of two degrees of freedom such as a violation of a Bell-like inequality [8], a Kochen–Specker-like phenomenon [9,10] and a quantum state tomography [11]. After the successful implementation of a coherent energy manipulation of neutrons [12] together with zero-field and Larmor spin manipulations [13], we proceeded to deal with further entanglement of three degrees of freedom, e.g., striking violation of a Mermin-like inequality [14] and a contradiction with a Greenberger–Horne–Zeilinger-like (GHZ-like) state [15] are confirmed [16]. In addition, entanglement between energy and spin enabled to implement higher-fidelity and more reliable manipulation of the entanglement of two degrees of freedoms of single-particles in neutron polarimeters [17].

2. Entanglement of two degrees of freedoms

The total state of a neutron is described by a tensor product of mutually disjoint Hilbert spaces, i.e., $H = H_1 \otimes H_2 \otimes \ldots \otimes H_n$, where H_j can be spinor, spatial, energetic Hilbert spaces. In the following four experiments, the first three deal with $H = H_{\mathrm{spin}} \otimes H_{\mathrm{path}}$, and the last deals with $H = H_{\mathrm{spin}} \otimes H_{\mathrm{energy}}$: entanglement between two degrees of freedom of neutrons is considered here.

2.1. Violation of a Bell-like inequality

Bell's inequality for a bipartite entangled system is widely used to distinguish remarkable quantum non-local effects from

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(classical) local assumption. For entanglement between spin and path, the state of a neutron is prepared in a Bell-like state,

$$|\Psi_{Bell}\rangle = \frac{1}{\sqrt{2}}\{|\uparrow\rangle|I\rangle \pm |\downarrow\rangle|II\rangle\} \quad (1)$$

The (classical) non-contextual assumptions set a limit on a sum of expectation values: a Bell-like inequality is derived with four expectation values E

$$S' \equiv E(\alpha_1,\chi_1) + E(\alpha_1,\chi_2) - E(\alpha_2,\chi_1) + E(\alpha_2,\chi_2) < 2 \quad (2)$$

where α_j and χ_k are the spinor rotation angle and the phase shift tuned by an auxiliary phase shifter, respectively. In contrast, quantum mechanics predicts the value of $2\sqrt{2}$ for the Bell-like state.

The neutron interferometer was used to implement coherent beam handlings and spinor manipulations. The setup is depicted in Fig. 1. The sinusoidal intensity modulations were measured by tuning the spinor rotation angles to 0, $\pi/2$, π and $3\pi/2$ We obtained the final value $S' = 2.051 \pm 0.019 > 2$. This clearly shows a violation of the Bell-like inequality and confirms quantum contextuality [8]. The drop from the quantum prediction is solely due to the reduced contrast of the interference fringes.

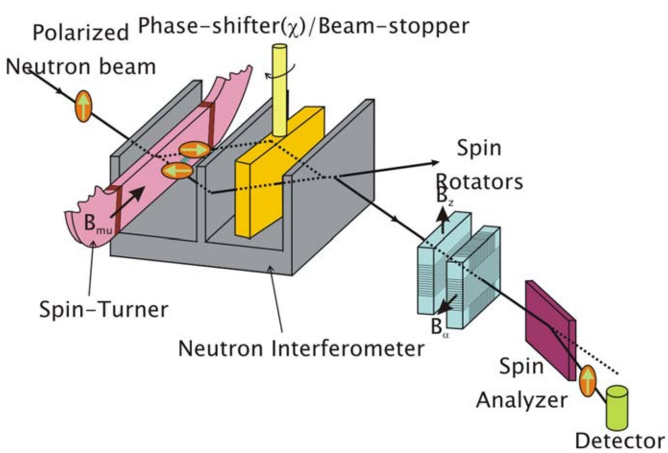

Fig. 1. Schematic view of the experimental setup with neutron interferometer to study the entanglement between spin and path degrees of freedom.

2.2. Kochen–Specker-like phenomenon

With the Bell-like inequality, one can study only the statistical violation of the (classical) non-contextual assumption. It was Kochen and Specker who gave a proof of the contradiction between quantum contextuality and the classical non-contextual assumption [9]. Based on the interference effect of matter waves, neutron interferometric experiments are suitable to exhibit a phenomenon associated with the Kochen–Specker theorem.

The setup of the experiment is practically the same as the first experiment (see Fig. 1). The Bell-like state is prepared and suitable product observables were measured. The obtained values exhibit a contradiction between standard quantum mechanics and the non-contextual theories by a fraction as large as 63% (ideally 100%!). The reduction is solely due to about 65% contrast of the interference fringes from the interferometer. In order to analyze the result quantitatively, we derived two inequalities, both of which again exhibit clear violation of the (classical) non-contextual assumption [10].

2.3. Quantum state tomography

For a thorough characterization of a superposed and entangled system, a complete set of tomographic measurement is required: a density-matrix formula for the tomographic reconstruction contains all the physical information about a quantum ensemble. We have achieved such a complete tomographic measurement of our non-separable Bell-like states.

In the experiment, a similar setup was used to the previous experiments (Fig. 1). A tomographic measurement of a quantum system ρ_{Bell}, consisting of two-qubits, requires a full set of 16 operators. For convenience, we chose the observables, $\hat{O}_j = \{\hat{P}^p_{+x}, \hat{P}^p_{+y}, \hat{P}^p_{\pm z}\} \otimes \{\hat{P}^s_{+x}, \hat{P}^s_{+y}, \hat{P}^s_{\pm z}\}$ $(j = 1, \ldots, 16)$, where $\hat{P}_k$ are projection operators to k-directions, p and s represent path and spin components. Some of the obtained interference oscillations indicate complementary intensity dependence, which are attributed to the entanglement between the spinor and the spatial wave function in single neutrons. A reconstructed density matrix is shown in Fig. 2. The fidelity $F \equiv \langle \Psi_{Bell}|\rho_{Bell}|\Psi_{Bell}\rangle = 0.79 \pm 0.03$ and the concurrence (a measure of degree of entanglement) $C = 0.59 \pm 0.03$ were obtained [11]. The drop here results mainly from the reduction of the contrast of the interference fringes.

2.4. Energy-spin entanglement

In the three experiments above, entanglement was achieved between the spin and the path degrees of freedom. Recent

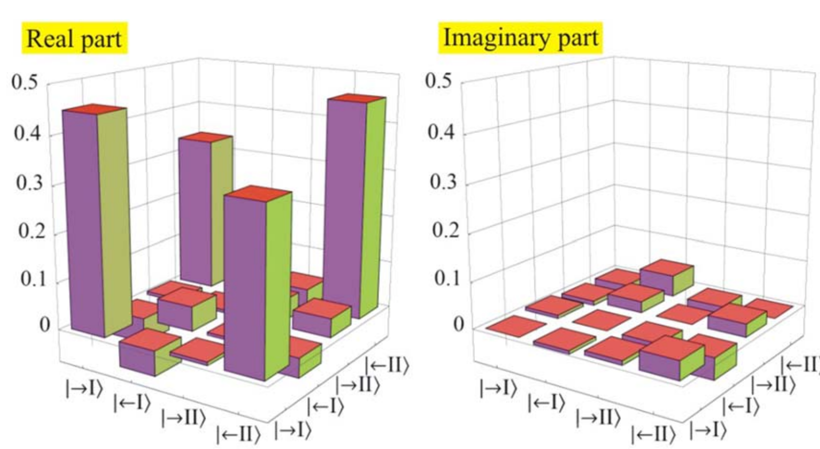

Fig. 2. Result of the state tomography of the Bell-like state consisting of neutron's spin and path degrees of freedom.