X-ray spectra reconstruction from analysis of attenuation data: A Back Scattering Thomson source application

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ABSTRACT

It is known that useful information about the spectral distribution of an X-ray beam can be extracted by measuring and analyzing its transmission curve. This kind of approach to the characterization of the distribution in energy of photons is justified when the direct measurements of the energy of a single photon becomes too expensive or cannot just be performed. Thomson Scattering sources can produce up to 10^8 photons within a pulse 10 ps long. Hence the rate of incidence of photons is too high to apply traditional X-ray spectroscopy methods while methods based on the transmission curve can still be used.

In order to propose an application of these techniques in the characterization of a Thomson Scattering source we studied an iterative statistical algorithm (Expectation-Maximization) used as a regularization method on simulated measurement computed starting from a calculated energy distribution with peak energy of 20 keV. Results show that this method can give good approximations in the low energy range (approximately 20 keV) and that it is at least sensible to a small amount of radiation in the higher part of the energy range (approximately 70 keV). The robustness of the method against non-ideal experimental conditions is also considered.

1. Introduction

Thomson Back Scattering sources can be bright X-ray sources that typically produce photons in a pulsed modality, this because laser light and electrons are bunched before the collision. There are some working Thomson Back Scattering sources (also called Inverse Compton Scattering, depending upon electron and produced photon energies), for example at BNL (Brookhaven National Laboratories) [1] and University of Tokyo [2], while some other are under construction, for example at Daresbury [3] or at LNF (National Laboratories of Frascati) [4,5].

For the Thomson Back Scattering source at LNF are expected 10^9 photons per second, bunched in 10 ps long pulses at a repetition rate of 10 Hz with 8 mrad of divergence [6]. In this condition the rate of fluence (photon per second per square millimeter) is too high to perform a standard spectrometric measurement based on single photon energy measurement.

Anyway, the knowledge of the spectrum of an X-ray source is a key point for the development of any kind of application, for example in imaging both contrast and absorbed dose strongly depend upon energy. Moreover, the energy distribution of the emitted photons is the final probe to check if the machine is correctly working. An alternative way to measure the spectrum of an X-ray source might be of interest in the experimental characterization of this kind of sources, and a method that requests the measurements to be integral-type will not be affected by the high rate of incidence of photons.

The analysis of attenuation data (transmission curves), for example exposure or dose measurement with an ionization chamber, can provide some information about the spectral distribution of an X-ray source and, as not affected by the rate of incidence of photons, is a good candidate for the characterization of a Thomson Back Scattering X-ray source.

2. Materials and methods

The exposure *Exp* is related to the spectrum of an X-ray beam in the following way [7]:

$$Exp = k \int_0^\infty \Phi_E(E) \mu_{tr} E \, dE \tag{1}$$

where $\Phi_E(E)$ represent the spectral distribution of the photons, μ_{tr} is the mass transfer coefficient of air and $k = (0.00873)^{-1} \, J^{-1} \, kg \, R$ is the constant to convert the kerma in air to exposure [8]. With an

ionization chamber it is possible to measure the exposure as a function of the thickness t of a well characterized material (for example Aluminium) used to filter the beam. For a perfectly collimated X-ray beam, the exposure is related to the thickness t of the absorber by the relation

$$Exp(t) = k \int_0^\infty \Phi_E(E) e^{-\mu(E)t} \mathscr{Z}(E)\, dE \qquad (2)$$

where $\mathscr{Z}(E) = \mu_{tr}(E)E$, which is considered a known function of energy, and $\mu(E)$ is the total attenuation coefficient of the attenuator in which are included both photoelectric and Compton effects. Each scattered photon is considered removed from the beam, as well as the photons absorbed by photoelectric effect.

In general, there is no analytical expression for the distribution of photons as function of energy, one choice could be to discretize the integral Eq. (2) and treat the problem from its numerical point of view; it turns out a linear system of the form

$$T = AX. \qquad (3)$$

Now $T \in \mathbb{R}^M$ represent the attenuation curve (simulated exposure measurements as a function of the thicknesses of the attenuator) and $X \in \mathbb{R}^N$ is the discretized spectrum. Both these functions are represented by vectors and $A \in \mathbb{R}^{M \times N}$ is a matrix which contains the properties of the attenuators. Matrix elements are defined as $A_{mn} = e^{-\mu_n t_m}$ where μ_n is the total attenuation coefficient of the attenuator computed at the energy corresponding to the n-th energy bin of the spectrum and t_m is the thickness of the attenuator related to the m-th transmission measurement ($m = 1 \ldots M$ and $n = 1 \ldots N$). The vector X can be regarded as the point-by-point product of S and Z which represent, respectively, the discretization of Φ_E and $\mathscr{Z}$ (Eq. (2)). Once X is computed it is possible to correct for the coefficients contained in Z and a spectrum in terms of number of photons per energy bin is obtained. Through this paper, which deals with a simulation study, we refer to the vector T as the (simulated) measurements considered as composed by an exact term T^0 and a random perturbation e representing the noise inherent in the measure process

$$T = T^0 + e \qquad (4)$$

in our case e is artificially added to T^0 in order to simulate a real experimental situation. This separation between the exact term T^0 and its inherent noise e is an idealization for clearness purposes: strictly speaking, when dealing with an ill-posed problem, even the roundoff error due to discretization in the computer acts like a critical noise contribution. In this sense the two terms are never separable one to each other.

The compactness of the linear operator described by Eq. (2) translates, after the discretization, into an ill-conditioning of the linear system in Eq. (3). This means that some regularization technique must be used to obtain a solution of the system $X = A^{inv}T$ which contains some useful information. We use Expectation-Maximization (also known as Richardson–Lucy) algorithm to obtain an approximation to the exact solution of the system of Eq. (3) from the measurements vector T, a description of this algorithm can be found in Appendix A.

We consider a calculated spectrum for a Thomson Back Scattering source (solid line in Fig. 2, the plots refer to a single pulse spectrum) [6]; starting from this distribution we compute the exact attenuation curve with Eq. (3) (Fig. 1). In [6] the spectrum is computed with an analytic code in the energy range 10–65 keV and the spectrum is discretized using 0.5 keV energy bins. The attenuator is Aluminium and the thicknesses are between 0 and 1.6 cm (the system matrix is 139×139). The attenuation coefficient are computed on the basis of the method proposed in [9]. Once the exact curve is computed the noise is
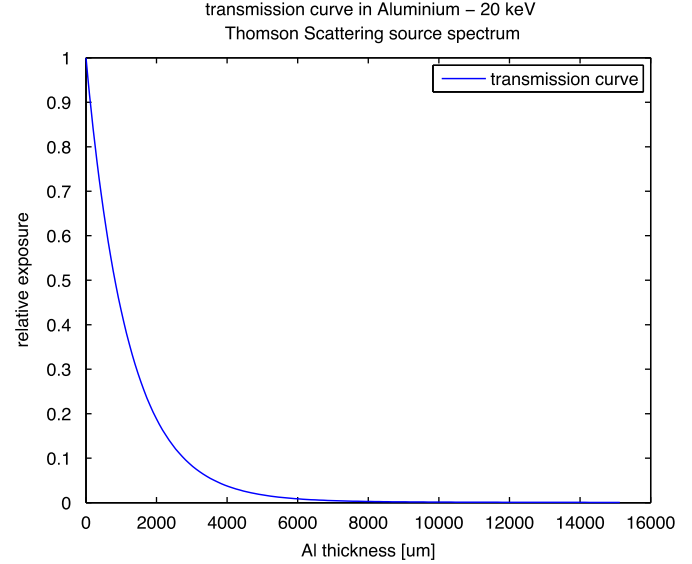
Fig. 1. Transmission curve computed for the reference 20 keV Thomson Back Scattering source spectrum (Fig. 2) using Aluminium as attenuator and an ionization chamber as detector. The beam is filtered with different thicknesses of Al ranging between 0 and 1.6 cm; the values of the exposure are normalized such that the exposure associated with the unfiltered beam is 1.

added. We consider three different levels of this perturbation, extracted from a random Normal distribution with sigma, respectively, 10^{-3}, 5×10^{-3} and 10^{-2} times the value of the exposure measurement. These values are chosen considering reasonable numbers for a real experiment. The Expectation-Maximization algorithm is then applied to the perturbed curves, since it is iterative we have to choose the input distribution and we need a stopping rule.

As there are no simple constraints to impose on the spectral distribution of Thomson Back Scattering sources we use a flat distribution as the input spectrum. The total area is set to be comparable with the first attenuation measurement (no filtration) and then the value of each bin can be easily computed, in other words, the first point of the attenuation curve is the total number of photons weighted with $\mathscr{Z}(E)$, the area under the flat distribution is set to be comparable to this number.

To find the number of iterations to be performed we use the Discrepancy Principle [10,11]; the basic idea is to consider that we cannot expect that the transmission curve, computed using an approximation to the spectrum extracted from the measurements, is closer to the exact transmission curve than the measured one. Consider the quantity

$$r(k) = \|T - AX^k\|_2 \qquad (5)$$

that we call residual and which is function of number of iterations k; T is the vector representing the measurements and X^k is the approximation obtained as the output of the k-th cycle of the algorithm. The symbol $\|\cdot\|_2$ represent the Euclidean norm, i.e. $\|T\|_2 = \sqrt{\sum_i T_i^2}$. Since we are interested in the distance between the measured attenuation curve T and the one computed using the k-th reconstructed spectra, the Euclidean norm (also called L_2 norm) is suitable as a quantification of this feature. This distance is the mean square error we commit if we take the AX^k as an approximation of T [12]. Recall Eq. (4), we can define the threshold $d = \|e\|_2$. Now the Discrepancy Principle states that the iteration has to be stopped when the condition $r(k) \le d$ is verified.