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Charge collection efficiency in ionization chambers operating in the recombination and saturation regimes

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ABSTRACT

We solve the electric charge transport equations in the recombination and saturation regimes using an iterative perturbation method. We then calculate the charge collection efficiencies of ionization chambers. The formulae obtained are presented in the form of series for which we calculate the first coefficients. Our approach allows to account for the spatial as well as the temporal variations of the primary charge density $N(\underline{r},t)$ in the calculations. Finally, we apply our method to study different density patterns, N, including the textbook case $N = N_0\delta(t)$ and the charge clusters and columns.

1. Introduction

In a preceding article [1], we studied ionization chambers operated in current mode, in order to calculate the shapes of their saturation curves and evaluate the densities of charge carriers in their filling gases/liquids. We observed that the behaviour of this class of detectors is governed by a system of basic but non-linear equations (see for instance Refs. [2–5]). We noted that, in the specific case where the space charge perturbations are neglected and the primary charge density induced by the ionizing particles is considered uniform, this system possesses an analytical solution. However, we noted also that this is in fact an exception, and the complete resolution of the problem still appeared out of reach. Consequently, ionization chamber operators had to resort to approximate or numeric solutions to optimize their detectors' characteristics and analyse their data.

To overcome this difficulty, we developed a mathematical method to solve the steady-state charge transport equations by iterations [1]. This approach allowed us, for the first time, to account for the spatial variations of the ionizing dose rate in the calculations. Moreover, we found that this mathematical tool could help resolve a related but more complex problem, the prediction of the charge collection efficiency in ionization detectors functioning in quasi-pulse mode. In this operating mode, the primary charge density induced by the ionizing particles in the filling fluid can vary with time as well as with the spatial coordinates. Time derivatives thus appear in the equations, complicating their resolution even more. As a result, a number of authors investigated a common sub-case, where the ionizing dose was considered instantaneous and uniform (see Refs. [6–8] for instance). In this framework, our predecessors obtained useful analytical formulae, among which we can quote the reference result of Boag for parallel plane chambers [6]:

$$\frac{Q}{Q_0} = \frac{\ln(1+\xi)}{\xi}, \quad \xi = \frac{kN_0 d}{(\mu_e + \mu_a)E} \tag{1.1}$$

where Q/Q_0 is the fraction of charge carriers that escaped recombination and reached the electrodes; N_0 is the initial density of electron–ion pairs created inside the filling gas/liquid; μ_e and μ_a are the electronic and ionic mobilities, E the electric field generated between the electrodes, k the recombination coefficient and d the inter-electrode spacing. For more general primary charge densities (non-uniform and non-instantaneous), however, no analytical results for the Q/Q_0 ratio are available in the literature to our knowledge.

To address this problem, we extend our perturbation method to the quasi-pulse operating mode, with, as our objective, a general solution of the ionization detectors' modelling equations in the recombination and saturation regimes. In the absence of space charges and attachment reactions of electrons over impurities or neutral atoms, these equations write:

$$\begin{cases} \frac{\partial n_e}{\partial t} - \mu_e\,\underline{E}\,.\,\underline{\nabla}\,n_e = N(\underline{r},t) - kn_e n_a \\ \frac{\partial n_a}{\partial t} + \mu_a\,\underline{E}\,.\,\underline{\nabla}\,n_a = N(\underline{r},t) - kn_e n_a \end{cases} \tag{1.2}$$

where N is the density of electron–ion pairs created per unit of time by the incident particles in the detector; n_e and n_a are, respectively, the electron and ion densities in the filling fluid.

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This system is completed by two boundary conditions: (1.3) the electronic density n_e is zero at the cathode surface, (1.4) the ionic density n_a is zero at the anode surface. Note that Eqs. (1.2) can be used for other charge carriers, such as positive and negative ions. Note also that, in (1.2) system, we neglect the diffusion processes. This approximation is valid only when the diffusion speeds of electric charges are negligible compared to their drift speeds in the electric field. Thus, for detectors operated at low voltages with high-density localized charge structures, the results obtained in this paper should be applied with caution.

A direct resolution of the problem $P=\{(1.2)+(1.3)+(1.4)\}$ for an unspecified density $N(\underline{r},t)$ seems impossible at first sight. Consequently, in Section 2, we re-examine the operators D_e and D_a defined in [1] and demonstrate that they can be generalised to account for the temporal as well as the spatial variations of the ionizing dose $N(\underline{r},t)$ in the calculations. In Section 3, we apply the resulting formulae to predict the charge collection efficiency of ionization detectors for several primary charge density patterns, including the textbook problem $N=N_0\delta(t)$ and the charge cluster and columns.

2. Presentation of the perturbation method

To solve the problem $P=\{(1.2)+(1.3)+(1.4)\}$, we treat the recombination term kn_en_a as a perturbation and rewrite the (1.2) equations by introducing a perturbation coefficient ε:

$$\begin{cases} \dfrac{\partial n_e}{\partial t} - \mu_e\,\underline{E}\,.\,\underline{\nabla}\,n_e = N(\underline{r},t) - \varepsilon k n_e n_a \\ \dfrac{\partial n_a}{\partial t} + \mu_a\,\underline{E}\,.\,\underline{\nabla}\,n_a = N(\underline{r},t) - \varepsilon k n_e n_a \end{cases} \tag{2.1}$$

$$\text{with}\begin{cases} n_e = N_e^{(0)} + \varepsilon N_e^{(1)} + \varepsilon^2 N_e^{(2)} + \cdots + \varepsilon^n N_e^{(n)} + \cdots \\ n_a = N_a^{(0)} + \varepsilon N_a^{(1)} + \varepsilon^2 N_a^{(2)} + \cdots + \varepsilon^n N_a^{(n)} + \cdots \end{cases} \tag{2.2}$$

In parallel plane chambers (cf. Fig. 1), the electric field strength is given by

$$\underline{E} = \frac{\Delta V}{d}\,\underline{u}_x \tag{2.3}$$

if we neglect the space charge perturbations and the finite size effects. d is the inter-electrode gap and ΔV, the voltage applied at the electrodes. The (1.3) and (1.4) boundary conditions imply:

$$n_e(x=d,y,z,t) = n_a(x=0,y,z,t) = 0,\ \forall(y,z,t) \tag{2.4}$$

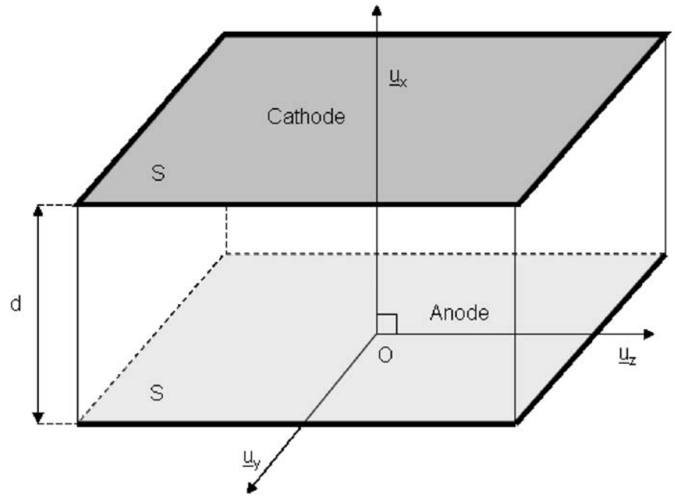

Fig. 1. A schematic view of a parallel plane ionization chamber.

Integrating the (2.1) equations with the (2.4) conditions, we obtain:

$$\begin{cases} n_e(x,y,z,t) = \frac{d}{\mu_e\Delta V}\int_{x'=x}^{d}\Big[N\Big(x',y,z,t+\frac{x-x'}{v_e}\Big) \\ \qquad -\varepsilon k n_e n_a\Big(x',y,z,t+\frac{x-x'}{v_e}\Big)\Big]\,dx' \\ n_a(x,y,z,t) = \frac{d}{\mu_a\Delta V}\int_{x'=0}^{x}\Big[N\Big(x',y,z,t-\frac{x-x'}{v_a}\Big)\,dx' \\ \qquad -\varepsilon k n_e n_a\Big(x',y,z,t-\frac{x-x'}{v_a}\Big)\Big]\,dx' \end{cases} \tag{2.5}$$

where $v_e=\mu_eE$ and $v_a=\mu_aE$ are the electronic and ionic drift speeds. As we did in Ref. [1], we introduce two operators, D_e and D_a, to simplify the intermediate calculations:

$$\begin{cases} D_e: f(x,y,z,t) \to \int_{x'=x}^{d} f\Big(x',y,z,t+\frac{x-x'}{v_e}\Big)\,dx' \\ D_a: g(x,y,z,t) \to \int_{x'=0}^{x} g\Big(x',y,z,t-\frac{x-x'}{v_a}\Big)\,dx' \end{cases} \tag{2.6}$$

Using expressions (2.2), (2.5) and developing them as powers of ε, we obtain:

$$\begin{cases} N_e^{(n)} = -\frac{kd}{\mu_e\Delta V}D_e\left(\sum\limits_{u=0}^{n-1}N_e^{(u)}N_a^{(n-1-u)}\right) \\ N_a^{(n)} = -\frac{kd}{\mu_a\Delta V}D_a\left(\sum\limits_{u=0}^{n-1}N_e^{(u)}N_a^{(n-1-u)}\right) \end{cases} \text{for } n\geqslant 1 \tag{2.7}$$

$$\text{with}\begin{cases} N_e^{(0)} = \frac{d}{\mu_e\Delta V}D_e(N) \\ N_a^{(0)} = \frac{d}{\mu_a\Delta V}D_a(N) \end{cases} \tag{2.8}$$

We note that the $N_e{}^{(0)}$ and $N_a{}^{(0)}$ terms are the charge densities obtained in the absence of recombination reactions. By making the perturbation coefficient ε approach 1 in (2.2), we obtain the limited developments of the densities, n_e and n_a, as powers of $1/\Delta V$:

$$\begin{cases} n_e = N_e^{(0)} + N_e^{(1)} + N_e^{(2)} + \cdots + N_e^{(n)} + \cdots \\ n_a = N_a^{(0)} + N_a^{(1)} + N_a^{(2)} + \cdots + N_a^{(n)} + \cdots \end{cases} \tag{2.9}$$

At the orders 1–3, the formulae of the densities, $N_e{}^{(n)}$ and $N_a{}^{(n)}$, are given in Ref. [1] and recalled in Appendix A.

We are now able to calculate the electric charge, q_e, collected per unit of time at the anode:

$$\begin{aligned} q_e(t) &= -ev_e\iint_S n_e(x=0,y,z,t)\,dy\,dz \\ &= -ev_e\sum_{n=0}^{+\infty}\iint_S N_e^{(n)}(x=0,y,z,t)\,dy\,dz \end{aligned} \tag{2.10}$$

where S is the anode surface. Using formulae (2.7) and (2.8), we can calculate the limited development of q_e as powers of $1/\Delta V$:

$$q_e(t) = -e\times\sum_{n=0}^{+\infty}(-1)^n\left(\frac{kd^2}{\mu_e\mu_a}\right)^n\frac{\gamma_n}{\Delta V^{2n}} \tag{2.11}$$

with γ_n at first orders

$$\gamma_0 = \iiint_V H(\underline{r},t)\,dx\,dy\,dz$$

$$\gamma_1 = \iiint_V D_e(H(\underline{r}',t))D_a(H(\underline{r}'',t))\,dx\,dy\,dz$$