Nuclear Instruments and Methods in Physics Research A 604 (2009) 684–693

Contents lists available at ScienceDirect

Nuclear Instruments and Methods in Physics Research A

journal homepage: www.elsevier.com/locate/nima

Empirical expressions for angular deviation of muons transmitted through slabs of iron, lead and uranium

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ARTICLE INFO

Article history:
Received 16 December 2007
Received in revised form
20 November 2008
Accepted 27 February 2009
Available online 11 March 2009

Keywords:
Cosmic rays
Muons
Radiography
Monte Carlo simulation
Empirical formulations

ABSTRACT

To facilitate the computation of angular deviation of cosmic-ray muons transported through slabs of iron, lead and uranium, we have developed an empirical relation for the angle of deviation with the incident muon energy and the thickness, atomic mass, radiation length and density of the material. This empirical relation can be used for easy calculation of angular deviation of cosmic-ray muons that are proposed for detection of high-*Z* materials in the presence of materials of low-*Z* using the muon radiography technique. Our results indicate that uranium and lead can be distinguished from iron while discrimination between lead and uranium is anticipated to face practical difficulties.

1. Introduction

Recently an interesting development has taken place in the field of cosmic-ray muon radiography [1,2]. In this technique cosmic-ray muons are used for image reconstruction and material *Z* discrimination. Experimental results and theoretical simulations have demonstrated the ability to reconstruct complex objects and to detect materials of high atomic number hidden in a much larger volume of low atomic number material using cosmic-ray muons. It is claimed that successful application of this technique can detect $9\times9\times12\,\mathrm{cm}^3$ of uranium buried in a $6.4\times2.4\times2.4\,\mathrm{m}^3$ cargo container having 12 ton of distributed iron. This method is anticipated to be very useful in surveillance for cross-border transport of nuclear and other heavy materials.

Due to large penetrating power of high-energy muons available in cosmic-ray shower, muons are more useful for radiography of thick objects than X-rays or gamma-rays. Also, one can avoid the unnecessary radiation hazard caused by X-rays or gamma-rays. Due to multiple Coulomb scattering, muons, while passing through any thick object, deviate in angle and the deviation depends on the atomic number *Z* and density of the material. The flux of cosmic-ray muons reaching the surface of the Earth is about 10,000 muons per minute per square meter. The mean energy of muons at sea level is about 3–4 GeV. These high-energy muons having large penetrating power provide a way of discriminating materials of different densities.

Muons passing through a material lose energy through different processes. The main processes of muon energy loss are: atom excitation and ionization (i.e. collision losses); bremsstrahlung, electron–positron pair production and inelastic scattering (photonuclear interaction). Collision losses are described by cross-sections comparable to atomic dimensions and occur very frequently along the path of the ion. But in the muon energy range considered here, since energy losses per collision are small (about 10 eV), the resulting angular deviation per collision is also very small. This process of multiple scattering does not contribute much to the muon energy loss, so it is usually neglected in considering muon energy degradation. The other three interactions occur when the muon passes close to the nuclear protons and hence these are relatively rare, but since they involve much larger energy losses (about a few MeV) the muon scattering angle is quite large even in a single event. The mean free path for these large momentum transfers is called the radiation length which decreases with increasing atomic number and density. These processes produce muon deflection from the initial direction and the resultant scattered angle depends on the radiation length of the material.

A widely used theory of multiple scattering of charged particles in the Coulomb field of nuclei is due to Moliere, especially in the form given by Bethe [3]. A comprehensive comparison of the Moliere theory with experimental data on multiple scattering of 1 MeV to 200 GeV protons shows that this theory, with some corrections, is accurate to better than 1% on

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doi:10.1016/j.nima.2009.02.037

average except for thick absorbers [4]. The Moliere theory is based on a single scattering cross-section, which has infinite moments. Therefore, this theory has a wrong asymptotic at large thicknesses [5]. The angular distribution of transported muons can be approximated as a Gaussian with a long tail [6]. With the Moliere prediction of the $1/e$ angle and assuming the distribution of angular deviation to be a Gaussian, the width of the distribution is given by [6]

$$\sigma_M = \frac{13.6\,(\mathrm{MeV})}{\beta cp}\sqrt{t/L_R}[1 + 0.038\ln(t/L_R)]. \quad (1)$$

Here p and βc are the momentum and velocity of the incident muon, and t/L_R is the thickness (t) of the scattering medium in radiation lengths (L_R). This value of σ_M is from a fit to Moliere distribution for singly charged particles with $\beta = 1$ for all Z, and is accurate to about 11% [6].

Radiation length, the characteristic range for scattering and other electromagnetic interactions, decreases with increasing atomic number Z of the material. The mean scattering angle for a given material depth therefore increases with Z. This Z sensitivity coupled with the long range of muons makes muon scattering of particular interest as an information source for the detection of high-Z materials in low-Z surroundings. Jenneson et al. [7] have carried out some measurements and have proposed a technique of imaging the internal structure of large vessels by using cosmic-ray muon energy loss estimations. These authors have indicated that the energy loss measurement technique is a promising one and can be extended for material identification though the proposed method faces several practical difficulties of muon energy measurements before and after propagation through the material which is further complicated by the cosmic-ray muon energy distribution.

The main aim of the present work is to establish an empirical expression to estimate the angular deviation of muons transported through thick slabs of uranium, lead and iron with reasonable number of adjustable parameters. Additionally, the empirical expression should relate the angular deviation with the incident muon energy, thickness and other physical properties of the material. It can be safely assumed that if uranium and lead can be discriminated from iron, then they can be discriminated from lower Z materials also. Such an empirical expression will help computer aided muon radiography work, since large number of quick calculations of the angular deviation would be required for different thicknesses and interpolations in between. Finally, based on our empirical expression, we proposed a fast and easy method of discriminating high Z materials like uranium and lead from iron and other lower Z materials. In practical muon radiography, e.g. for scanning cargo, quick processing of data is of prime importance.

In this paper we present the results of Monte Carlo (MC) simulations to estimate the angular deviation from the incident directions (not angular distribution) of muons transported through different materials like U, Pb and Fe. Fig. 1 gives a schematic view of the multiple scattering of muons through a slab and the resulting angular deviation at the exit surface. Thereafter, we have obtained an empirical relation by correlating these distributions with the characteristics of the materials and the energy of the incident muons. The empirical relation is expected to provide a way of easy and quick estimations of angular deviations of transported muons that can be used to detect high Z materials in presence of low Z surroundings. Such calculations, using detailed MC simulations, are rather time consuming and involve non-trivial efforts in input preparation. For MC simulations we have used the code MUSIC (Muon Simulation Code) [8], which has been developed to calculate energy distributions of muons at large depths underground. In the present work we have used a version developed particularly for relatively thin objects.

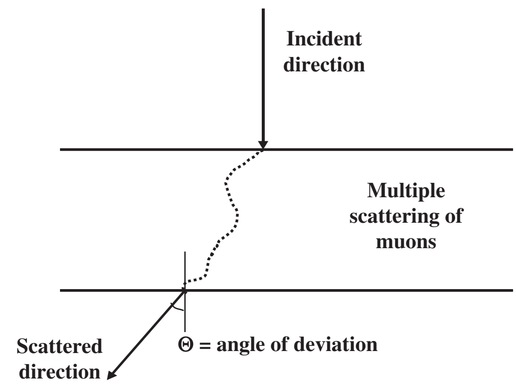

Fig. 1. Scattering of muons through material and angle of deviation at the exit surface. The magnitude of scattering is exaggerated for this illustration.

Commonly, MC simulation technique is used for muon transport since analytical solutions can at best be approximate due to the complexity of the problem and particularly because of the stochastic nature of muon energy losses. Such MC algorithms necessarily incorporate some uncertainties into the final result. Apart from the insurmountable uncertainties of input data, which relate to finite accuracy of formulae for muon cross-sections and data on medium density and composition there are other errors that are produced by simulation algorithm itself due to finite accuracy of numerical procedures adopted and simplifications that are done to get a reasonable computation time. Any muon MC propagation algorithm consists of a set of procedures on numerical solution of equations, interpolation and integration. All these procedures are of finite accuracy and, consequently, the incoming model for muon interactions is somewhat corrupted by them. Sokalski et al. [9] have studied different codes for evaluating the accuracy of muon transport simulation and have concluded that the optimum value for inner accuracy of a MC code for the muon transport is close to 1% and accordingly, the code MUSIC can be accepted as accurate enough. Additionally, the results of angular deviations calculated using the code has been compared with Moliere theory and only a small difference was found [8].

In Section 2 we give our computational procedure, describe briefly the code MUSIC and present our simulation results. In Section 3 we describe the procedure of fitting the estimated angular deviations to obtain the required empirical relation. In Section 4, we validate our empirical expression and try to find out the limit of discrimination among iron, lead and uranium slabs of different thicknesses using cosmic-ray muons.

2. Computational procedure

We have carried out MC simulations of muon transport through slabs of different materials to estimate the angular deviation of muons due to multiple scattering. We have considered slabs of uranium, lead, iron of different thicknesses ranging from 0.1 to 0.5 m with a step of 0.1 m. Parallel monoenergetic beams of muons of different energies in the range 1–5 GeV with a step of 1 GeV, incident normally on the slabs, have been considered. Our aim was to construct Green's function using the set of results obtained with monoenergetic beam so that for any measured spectrum of cosmic-ray muons we can obtain the spectrum of muons beyond the material using this function. We have used the MUSIC [8], in particular the version developed for muon propagation through small thicknesses of materials [10].