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Estimation of the mass of the lighter particle emitted in a two body decay reaction

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ABSTRACT

The mass of an elementary particle, produced in a two body decay, can be calculated by knowing the masses and by measuring the momenta of the decaying particle and of one of the two final state particles with its angle of scattering. In this note we describe the technical results of a Monte Carlo simulation code used for the analysis of the behaviour of the upper limit values at 90% confidence level of the mass of the unknown particle as a function of the experimental uncertainties of the measured momenta and angle. The results are useful in designing experiments aimed at the determination of the mass of an unknown light particle.

1. Introduction

The determination of the mass m of a particle produced in a two body decay reaction, when two of the particles are kinematically and dynamically defined, is a well-known procedure in particle physics [1].

With the present attainable experimental precisions in the measurements of momentum and angle, the determination is easy when the mass m is of the order of magnitude of the masses of the two other participating particles. A single measurement of the decay gives for the m^2, i.e. the square of the unknown mass, a positive value. Repeated measurements of the decay provide a statistic of positive m^2 values, in general distributed with Gaussian shape. In this case all the elements of the distribution reside in the physical region. The value of the unknown mass is calculated extracting the square root of the mean value of m^2.

When the mass of the light particle is one or more orders of magnitude smaller than those of the two other particles, the experimental measurement turns out to be a difficult task [2]. The combined effects of the smallness of the mass and of the magnitude of the experimental uncertainties of the measured quantities, sometimes, generate a negative value of m^2. As a consequence, in repeated measurements, the m^2 distribution has values in the non-physical region. In these conditions statistical procedures allow to estimate only the upper limit value of m, at a given confidence level [3]. Accurate and precise experimental measurements of all the kinematic and dynamic parameters of the decay are required.

When the values of the m^2 distribution reside totally in the non-physical region, no known statistical procedure exists for utilising the experimental data.

In this paper we report the results of a Monte Carlo simulation procedure we have performed to study the relevance of the experimental uncertainties of the measured quantities of the decay, in the estimation of the unknown mass m. In particular, the simulation shows, for the first time, the behaviour of the upper limit, and its factorisation terms, for the mass of an unknown elementary particle involved in a two body decay, as a function of the experimental uncertainties. The results extend the study on the upper limit value of a light particle mass we have performed measuring the momenta and the angle of scattering of the unique completely measured $\pi^+ \rightarrow \mu^+ \nu_\mu$ decay [4]. The simulations are performed at the same energy and scattering angle values. For the first time, starting from completely known two body decays of elementary particles, i.e. $\Lambda \rightarrow p\pi^-$,$\Phi \rightarrow K^+K^-$, $\Sigma^+ \rightarrow p\pi^0$, the simulation has been extended down to a

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muon-neutrino mass of 100 eV for the decay $\pi^+ \rightarrow \mu^+ \nu_\mu$. The results show, within the wide mass range from 500 MeV down to 100 eV, that the behaviour of the mass upper limits follows a power law as a function of the experimental uncertainties. The simulation we present will also be used to study the $\pi^\pm \rightarrow \mu^\pm \nu_\mu(\bar{\nu_\mu})$ decays for low and very low pion momentum in the PAINUC experiment [7]. The decays occur in a 1 atm helium filled streamer chamber exposed to the JINR phasotron pion beam in Dubna. This experiment should improve the result obtained with a complete $\pi\mu e$ decay chain observed at PS179 at CERN in a $\bar{p}Ne^{20}$ annihilation event [4].

In Section 2 we describe the two body decay reactions analysed. Section 3 shows the kinematic relations used. Section 4 reports, briefly, on the statistical estimator of a physical parameter and on the procedure to calculate its upper limit value. In Sections 5 and 6 we describe the procedure to construct the kinematic database for the simulation, and the Monte Carlo code, respectively. Section 7 contains the results of the simulated decays, and the discussions of the behaviours of the upper limit values as function of the experimental uncertainties. Section 8 contains the conclusions.

2. The two body reactions studied

The study has been divided into three parts. In the first, we have analysed the reactions

$$\Phi \rightarrow K^+ + K^- \quad (1)$$

$$\Lambda \rightarrow p + \pi^- \quad (2)$$

$$\Sigma^+ \rightarrow p + \pi^o. \quad (3)$$

The masses of all the particles are known. The K^- mass is about half of the Λ. In reactions (2) and (3) the pions have masses one order of magnitude lower than the heavier ones. The analysis of the behaviour of the distributions of m_{K^-}, m_{π^-} and m_{π^o} values, studied as a function of the experimental uncertainties of the momenta of Φ, Λ, Σ^+, K^+ and p and of the scattering angle between them, provides the test of the simulation code and information on the precisions of the measured physical quantities.

In the second part of the study we have analysed the two body decay

$$A \rightarrow B + X \quad (4)$$

where A, B and X indicate the particles of the decay. The masses of A and B have been set equal to the mass of Λ and of proton, respectively. X is the fictitious third particle. The X mass has been changed in each simulation. These decays have no counterparts in Nature. On the other hand, the energy–momentum conservation law allows the calculation of the momenta and the scattering angles of the particles of whichever decay. Likewise, with the Monte Carlo procedure it is possible to study the distributions of the kinematic and dynamic parameters of the corresponding X particle, as a function of the experimental uncertainties.

Finally, in the third part, we have applied the procedure to simulate the reaction

$$\pi^+ \rightarrow \mu^+ + \nu_\mu \quad (5)$$

where the masses of π^+ and μ^+ are known [1]. In the simulations we have taken into account five different values of the muon neutrino mass. Studying this decay, the very low mass region has been reached.

3. Two body decay kinematics

A two body decay reaction occurs in a plane [4]. Using the symbols of reaction (4), the direction of the decaying particle A makes an angle θ with the flight direction of particle B, and an angle ϕ with the flight direction of particle X. The relativistic conservation law of the energy and of the longitudinal and transverse momentum components allows to calculate the square of the mass of the X particle with the relation

$$m_X^2 = [(E_A - E_B)^2 - p_X^2] = (E_A - E_B - p_X)(E_A - E_B + p_X) = m^* m^{**} \quad (6)$$

where

$$p_X^2 = p_A^2 + p_B^2 - 2p_A p_B \cos\theta \quad (7)$$

is the square of the momentum of the particle X, m^* and m^{**} are two factors defined by

$$m^* = E_A - E_B - p_X \quad (8)$$

and

$$m^{**} = E_A - E_B + p_X. \quad (9)$$

E_A, p_A, E_B and p_B are the energy and the momenta of the particles A and B, respectively. The factor m^{**} is always greater than zero. The quantity m_X^2 has the same sign of the factor m^*.

If m^* is >0, then m_X^2 is >0 and the X particle mass is calculated extracting the square root of (6)

$$m_X = \sqrt{(E_A - E_B)^2 - (p_A^2 + p_B^2 - 2p_A p_B \cos\theta)} = \sqrt{m^* m^{**}}. \quad (10)$$

Eq. (10) shows clearly that m_X depends only on the kinematic quantities of particles A and B.

4. On the estimation of a parameter

The estimation of a parameter is performed by using a given number of experimental measurements. The estimate is a function of the data. It consists in determining a single value or a set of values by which one represents the true value of the parameter. The characteristics of an estimator are those described in books on statistical methods [5].

When repeated measurements are performed, sometimes, the calculated value of the parameter can be negative. If the parameter is the square of a physical quantity, the negative value resides in non-physical region. In our case the parameter to be estimated is m_X^2, and is calculated using Eq. (6). If the values are partially in the non-physical region, a classical statistical method allows to extract only a limited information on the value of the parameter.

Let us consider the m_X^2 parameter. Suppose the statistic is Gaussian and contains negative values of m_X^2, i.e. its distribution extends itself with a tail in the non-physical region of m_X^2. A classical confidence level p for m_X^2 can be selected whenever the corresponding classical confidence limit $m_{p,cl}^2$ is in the physical region. The square root of $m_{p,cl}^2$ is considered the upper limit of the parameter m_X at p confidence level. The upper p confidence limit is calculated with the equation

$$m_{p,cl}^2 = m^2 + Z_p \Delta m^2 \quad (11)$$

where m^2 is the mean of the distribution, Δm^2 is its standard deviation, and Z_p takes the values $Z_{68} = 1.0$, $Z_{85} = 1.036$, $Z_{90} = 1.282$ $Z_{95} = 1.645$ and $Z_{97.5} = 1.960$, at 68%, 85%, 90%, 95% and 97.5% confidence level, respectively [6]. The classical confidence limit satisfies the probability statement

$$P(m^2 < m_{p,cl}^2) = p \quad (12)$$