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Counting statistics distorted by two dead times in series which end with an extended type dead time

H.D. Choi *

Department of Nuclear Engineering, Seoul National University, Seoul 151-744, Republic of Korea

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ABSTRACT

The distorted counting statistics resulting from two dead times occurring in series are discussed. The cases studied are those of series combinations of non-extended–extended (NE–E) dead times and of extended–extended (E–E) dead times under a Poisson input distribution. Three choices of time origin of the counting processes are considered, leading to the distinct statistics of three distinct processes—ordinary, equilibrium, and shifted processes. A set of formulae is presented for the event interval densities, corresponding Laplace transformations, the expected number and variance of counts in a given duration and the associated asymptotic expressions. Results are validated by comparison with previously published Monte Carlo simulations and checking the mathematical expressions in certain reduction limits. A possible application of the derived formulae is discussed.

1. Introduction

The dead times (DTs) that occur in counting circuits are known not only to cause a loss of counts but also to distort the counting statistics, even if the original process can be described by a simple Poisson distribution derived from the exponential nature of event intervals. Since this topic is important to the experimenters in radiological science and in physics using accelerated beam, efforts to understand the effects of DT have long been made, and understanding of them has progressed by renewal theory and operational calculus [1–3]. The relevant mathematical works cannot be listed exhaustively, but it is easy to recommend some of the more comprehensive works [1–4]. The ideal classification of DTs into extended (E, also known as paralyzable or cumulative) types and non-extended (NE, non-paralyzable or non-cumulative) types can invariably be found in a typical modern textbook on radiation detection [5]. The E-type DT is initiated and extended for any arriving pulse by its magnitude τ, beginning from the instant of arrival. The NE-type DT is activated by an arriving pulse only when the pulse arrives at a time free from DT status, but the period of it is not extended by any pulse arriving during the DT period. All pulses arriving during the DT period are not allowed to pass (or register). In addition to E- and NE-DTs, pulse pile-up is another important source of counting loss. Pulse pile-up has compounded DT features depending on the details of the counting system. In a single channel counting with integral discrimination, pulse pile-up has the feature of E-type DT. In pulse height spectroscopy, pulse pile-up is further classified as leading edge pile-up or trailing edge pile-up. The trailing edge pile-up has the features of E DT. The leading edge pile-up has the effect of count loss from the first pulse's amplitude channel. When the pile-up rejection (PUR) technique is adopted at the input stage of analog-to-digital converter (ADC), the leading edge pile-up results in the loss of all counts involved in the pile-up from the spectrum. Pommé et al. demonstrated that the counting statistics of leading edge pile-up is fundamentally different from those of E- and NE-type DTs [6]. For a brief summary of studies concerning the effects of pile-up on counting, see Ref. [7]. In this study, the discussion is limited to E- and NE-type DTs. The DT of a single channel analyzer (SCA) or an ADC of fixed conversion time is usually described by NE type. Most detectors and amplifiers show pulse pile-up at high counting rates and hence their DT is usually assumed as the E type while some references classify the DT of most proportional detectors and scintillation counters as the NE type [8]. Little is known about the characteristics of count loss due to saturation effects in detectors and amplifiers under a very high counting rate situation. There is an interesting study which dealt with the saturation effect as a kind of E-type DT by using experimental and theoretical approaches to investigate the counting behavior of a scintillation detector in a high intensity gamma or neutron field [9]. In summary, it is not evident *a priori* which type of DT a particular detector or electronics element has.

After over a half-century's work by a few pioneering authors, the detailed statistics of counting are known for a single DT of either type under a Poisson input distribution [1–2,4,10–19]. A real counting system is usually composed of a detector and several electronic circuit modules combined in series, in parallel, or in both. Hereafter, the detector is regarded as a circuit element

* Tel.: +82 2 880 7205; fax: +82 2 889 2688.
E-mail address: choihdg@snu.ac.kr

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when considering the characteristics and effects of the DTs. Even if the type of DTs that occur in a circuit element can be supposedly classified according to the two idealistic types (E or NE), an exact analysis requires the general theoretical treatment of many DTs occurring in series or in parallel. To this author's knowledge, such an analysis has not yet been completed, a fact mainly due to complications involved in the relevant mathematics. For analysis of such a complicated DT system, the Monte Carlo simulation is used in practice because it is a versatile method. However, mathematical analysis would provide a firm basis for understanding the distorted statistics of the counting system and provide exact formulae useful for quick and convenient application. A theoretical study of two DTs in series was performed in even the earliest studies. The analysis done in such early studies focused mostly on computing the mean count rates or, more precisely, the asymptotic mean count rates [1,4,10,20–23]. There are four possible cases of two DTs combined in series; NE–NE, E–NE, NE–E, and E–E. The former two cases have been dealt with in previous studies [1,4,20,22,24], while, in the latter two cases, which end with an E DT, only the asymptotic mean count rates have been computed in early studies [1,4,10,20–23]. A full discussion of the interval distribution functions, and the exact mean and variance of counts, has been deferred primarily because of the complexity of the equations involved.

In this study, the event time interval density (TID) distribution , the exact mean and variance of counts in a measurement interval, and the associated asymptotic expressions are given in full set for the series of two DTs ended by an E-type DT, which are denoted by NE–E and E–E cases, respectively. These five statistics—TID and the exact and asymptotic mean and variance of counts in a measurement interval—comprise the observables which are directly measurable in the counting system and give information about the type and magnitude of DTs in the counting system. Three different methods of choosing the time origin, leading to the so-called "ordinary," "equilibrium" (or stationary) and "shifted" (or free counter) processes, have been considered, in accord to the summary given by Müller [18]. Most of the derivations are based on the existing formalism coming from renewal theory and operational calculus (and tempered by a patience for tedious algebra) [1–4,13–19]. Since some of the formulae encountered are of a quite lengthy and complicated form, they have not been listed here in full detail. For comparison and cross-check of the formulae derived here, we reference two published works [24,25], whose Monte Carlo results we also use to verify the developed expressions for the TID and variance of counting rates. The properties of the counting statistics are discussed for each case of the studied NE–E and E–E series DTs. A possible practical application of the derived formulae for future study is also discussed.

2. General considerations

A schematic diagram of the series combination of two DTs is given in Fig. 1. The basic quantities are the event TID $f(t)$ and the total density (or renewal density) $D(t)$ modified by the presence of the DTs in the circuit. The location of the circuit at which the densities are relevant is indicated by a left subscript, and the corresponding Laplace-transformed quantities are denoted by both a right superscript * and the variable s. The left superscript NE or E indicates the type of the *first* DT in the modified statistics. A second DT is meaningful only if it is larger than the first one ($\tau_2>\tau_1$), so this is the only case considered in the series model because a smaller second DT ($\tau_2<\tau_1$) has no effect on the overall counting. The originating distribution of event interval is assumed to be ${}_0f(t)=U(t)\rho\exp(-\rho t)$, a Poisson distribution with a constant

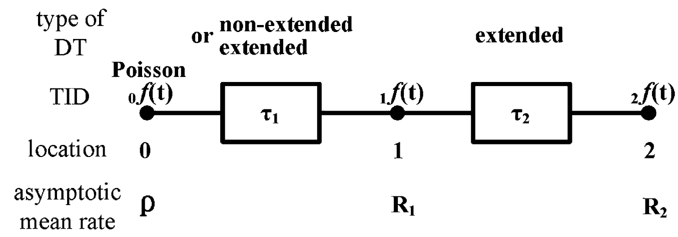

Fig. 1. A schematic block diagram of two dead times (DTs) in series. τ_2: DT of E type, τ_1: DT of either NE or E type. TID: time interval density distribution.

event rate ρ and where $U(x)$ is the Heaviside unit step function. The TID of pulse events modified by the first DT and its Laplace transform are well established and given by Refs. [1,2,4]:

$$ {}^{\mathrm{NE}}_{1}f(t)=U(t-\tau_1)\rho e^{-\rho(t-\tau_1)} \quad \text{and} $$
$$ {}^{\mathrm{NE}}_{1}f^*(s)=\frac{\rho e^{-\tau_1 s}}{s+\rho} \tag{1} $$

for the NE-type DT. For an E-type DT [2,4,13], the TID is given by

$$ {}^{\mathrm{E}}_{1}f(t)=\rho e^{-\rho\tau_1}\sum_{j=1}^{J_1}U(t-j\tau_1)(-\rho)^{j-1}(t-j\tau_1)^{j-1} \times e^{-(j-1)\rho\tau_1}/(j-1)! \tag{2} $$

where J_1 is the largest integer below t/τ_1, abbreviated as $[[t/\tau_1]]$. The corresponding transformed expression is

$$ {}^{\mathrm{E}}_{1}f^*(s)=\frac{\rho e^{-\tau_1(s+\rho)}}{s+\rho e^{-\tau_1(s+\rho)}}. \tag{3} $$

The multiple interval density of order k (or k-fold interval density) $f_k(t)$, the total density $D(t)$, and the corresponding Laplace transforms $f_k^*(s)$, and $D^*(s)$ are given in Refs. [1,2,4,13,16,17,22].

The equation that describes the effect of an E DT (the second DT in Fig. 1) with an arbitrary input distribution of pulse intervals is given in a compact form by Jost [1] as

$$ {}_2D(t)=U(t-\tau_2){}_1f(t)+\int_0^t {}_2D(t-\lambda){}_1f(\lambda)\,d\lambda \tag{4} $$

where $D(t)$ is the total density defined as the sum of all possible k-fold interval densities $f_k(t)$ ($k\geqslant 1$). The required density can be computed quickly using transformed quantities. Then, Eq. (4) is simply reduced to a relation in terms of the Laplace transformed quantities as:

$$ {}_2D^*(s)=L\{U(t-\tau_2){}_1f(t);s\}+{}_2D^*(s){}_1f^*(s) \tag{5} $$

where $L\{U(t-\tau_2){}_1f(t);s\}$ denotes the Laplace transformation of $U(t-\tau_2){}_1f(t)$ into the s-domain. Since the interval densities ${}_1f(t)$ and ${}_1f^*(s)$ are given in Eqs. (1)–(3) for the input distribution, ${}_2D^*(s)$ is obtained straightforwardly from Eq. (5) once the Laplace transformation of the term $U(t-\tau_2){}_1f(t)$ is performed for each case (NE–E or E–E series DT combinations) by direct integration or references to a table [26].

An equation equivalent to Eq. (4) may also be given in terms of the total density and interval density of the input as Ref. [1]:

$$ {}_2D(t)=U(t-\tau_2){}_1f(t)+\int_0^t {}_1D(\lambda){}_1f(t-\lambda)U(t-\tau_2-\lambda)\,d\lambda \tag{6} $$

which is useful for direct integration of the density functions ${}_1f(t)$ and ${}_1D(t)$. It is also useful for cross-checking the result obtained by inverse-transformation of ${}_2D^*(s)$ based on Eq. (5). Finally, the required interval distribution function ${}_2f(t)$ is obtained by an inverse transform of ${}_2f^*(s)$, which is related to the total density in the s-domain ${}_2D^*(s)$ as

$$ {}_2f^*(s)=\frac{{}_2D^*(s)}{1+{}_2D^*(s)}. \tag{7} $$