Nuclear Instruments and Methods in Physics Research A 597 (2008) 166–169

Contents lists available at ScienceDirect

Nuclear Instruments and Methods in Physics Research A

journal homepage: www.elsevier.com/locate/nima

Letter to the Editor

Track reconstruction using a 3-D map of the target magnetic field ☆

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ARTICLE INFO

Article history:
Received 16 July 2008
Accepted 4 September 2008
Available online 26 September 2008

Keywords:
Charged particles
Magnetic field
Relativity
Tracks

ABSTRACT

Magnetic fields are often applied to the target zone of a particle beam to maintain the polarization of gaseous targets, as with the HERMES experiment. The same field bends the trajectories of charged particles, however, introducing errors in vertex reconstruction.

This paper describes a method for accurately describing relativistic charged particle transport within a 3-dimensional (3-D), non-uniform magnetic field. The algorithm is tested on HERMES experimental data, and is shown to substantially improve the K_s^0 resonance in the $\pi^+\pi^-$ invariant mass spectrum. Indeed, corrected data taken with target magnet switched on are as good as data taken with magnet switched off. The method can easily be applied to other experiments given a 3-D magnetic field map of the target region. The relevant code is provided in an Appendix.

1. Introduction

The HERMES experiment (DESY laboratory, Hamburg) uses 27.6 GeV/c electrons or positrons from the HERA accelerator to study the quark-gluon spin structure of nucleons by deep inelastic scattering [1,2]. Thanks to the Sokolov-Ternov mechanism [3], the HERA beam is self-polarized transverse to its momentum. The beam polarization is rotated to the longitudinal direction at the HERMES interaction point by two spin rotators. Furnished with a dual-radiator Ring-Imaging Cherenkov detector (RICH) [4], the HERMES spectrometer is capable of distinguishing between pions, kaons and protons as well as characterizing several kinds of events.

HERMES employed polarized hydrogen and deuterium gas targets from 1996 to the end of 2005. A storage cell restricts the polarized atoms to a small volume in the path of the beam. The HERA beam passes through an open aluminum tube, with two side tubes for injecting the polarized gas and sampling its components after the interaction. The gas density inside the cell is about two orders of magnitude higher than a free jet would provide. The spin orientation of the target gas is defined by an external magnetic field. The field was applied parallel to the beam momentum until the end of 2000, then switched to a transverse orientation.

The external magnetic field is very important for two reasons: it determines the spin orientation of the target gas, and it effectively decouples the magnetic moments of electrons and nucleons. The latter is necessary to achieve a high degree of polarization in the injected gas and prevent spin relaxation. According to both theoretical and experimental results, the more intense the external magnetic field, the longer the target polarization can be sustained [5]. For longitudinal target polarization, the magnetic field is about 0.35 T. For transverse target polarization, the magnet design is more complex. A strong magnetic field is required for high polarization, but the intensity of the field is also limited by the amount of synchrotron radiation generated by beam deflection (5 kW maximum). After balancing these constraints, it was decided to use a field of about 0.3 T [5].

In principle, the external magnetic field will deflect all charged particles—not just the beam itself, but products of deep inelastic scattering. For this reason, the momentum measured by the HERMES spectrometer [6] (located downstream of the beam) is different from the particle's momentum at the relevant point of interaction or decay. (For example, a created particle may fly a measurable distance within the magnetic field before decaying.) The position of the interaction point cannot be obtained by extrapolating the straight lines provided by track detectors; the particles follow curved paths inside the target region, where no direct measurements are available. Not even the radius of curvature is constant in the target region, since the magnetic field is not uniform. As a result, the particle trajectory cannot be described by an analytic function. To learn the true momenta and positions of particles at their interaction points, we propose a method to obtain the actual trajectories of the charged particles from available data. This paper traces particles back to the target

☆ Supported by National Natural Science Foundation of China (10775006, 10375002 and 10675004), the Research Fund for the Doctoral Program of Higher Education (20070001008), and the China postdoctoral fund.

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doi:10.1016/j.nima.2008.09.007

given a three-dimensional (3-D) magnetic field map of the region and information provided by the HERMES spectrometer.

2. Charged particles transport within a magnetic field

The force $\vec{F}$ exerted by a magnetic field $\vec{B}$ on a particle with charge e and velocity $\vec{v}$ is given by the Lorentz formula:

$$\vec{F} = e\vec{v} \times \vec{B}. \tag{1}$$

A charged particle moving in a static, uniform magnetic field perpendicular to its trajectory experiences uniform circular motion with a radius ρ given by

$$\frac{1}{\rho} = \frac{qeB_i}{P_i}. \tag{2}$$

Here P_i is the (scalar) momentum of the particle and B_i is the intensity of the magnetic field. Note that the charge of the particle is given in units of e: $q = 1$ for a positron, π^+, K^+ or proton, and $q = -1$ for the corresponding antiparticles.

To learn the real trajectory of a low-momentum particle, its deflection due to the magnetic field must be known. Unfortunately, the real magnetic field is not uniform. For example, under traverse polarization the B_y component of the external field is dominant inside the target cell—but the B_x and B_z components also have to be taken into account for accurate experimental results.

Our method describes the transport of a charged particle from position $\vec{a}$ at time t to position $\vec{c}$ at time $t + \Delta t$. The relevant vectors are sketched in Fig. 1.

The magnetic field at $\vec{a}$ is denoted $\vec{B}(B_x, B_y, B_z)$. A particle at that position has momentum $\vec{P}$ in the laboratory frame. We define a new orthonormal coordinate system $\vec{u}$ as follows:

$$\begin{cases} \vec{u}_z = \dfrac{\vec{B}}{|\vec{B}|} \\ \vec{u}_x = \dfrac{\vec{P} \times \vec{u}_z}{|\vec{P} \times \vec{u}_z|} \\ \vec{u}_y = \vec{u}_z \times \vec{u}_x. \end{cases} \tag{3}$$

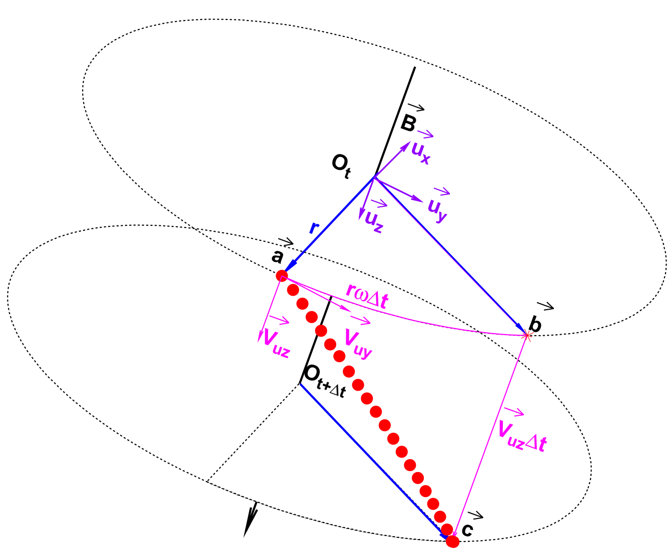

Fig. 1. Sketch of a charged particle deflected by a magnetic field $\vec{B}$. The red dots $\widehat{ac}$ trace the real trajectory from $\vec{a}$ at time t to $\vec{c}$ at time $t + \Delta t$. The algorithm consists of two steps: one to calculate the particle's revolution about O_t (from $\vec{a}$ to $\vec{b}$), and a second to calculate its linear motion along the field direction (from $\vec{b}$ to $\vec{c}$). See text for details.

The particle velocities in the $\vec{u}_z$ and $\vec{u}_y$ directions can then be written as

$$\begin{cases} V_{uz} = \dfrac{\vec{P} \cdot \vec{u}_z}{m} \\ V_{uy} = \dfrac{\vec{P} \cdot \vec{u}_y}{m} \end{cases} \tag{4}$$

where $m = \gamma m_0$ is the relativistic mass and m_0 is the rest mass. In this application the Lorentz factor γ can be defined conveniently as $E/(m_0c^2)$, where $E = \sqrt{|P|^2 + (m_0c^2)^2}$ is the total energy of the particle.

The velocity along the $\vec{u}_x$ axis is 0 by construction.

Assuming the magnetic field can be treated as uniform over a small volume near the current position of the particle, Eq. (2) gives the radius of curvature in the $\vec{u}_x - \vec{u}_y$ plane:

$$r = \frac{\vec{P} \cdot \vec{u}_y}{qe|\vec{B}|}. \tag{5}$$

The angular velocity of the particle about an axis located at point $O_t(r, 0, 0)$ in the $(\vec{u}_x, \vec{u}_y, \vec{u}_z)$ coordinate frame (see Fig. 1) is

$$w = \frac{V_{uy}}{r} \tag{6}$$

and the speed of the particle is

$$v = \frac{|\vec{P}|}{m}. \tag{7}$$

The time Δt required for the particle to move a preset distance length[1] l can be calculated as

$$\Delta t = \frac{l}{v}. \tag{8}$$

In the laboratory frame, the new velocity $\overrightarrow{O_tb}$ is created by rotating the vector $\overrightarrow{O_ta}(-r, 0.0, 0.0)$ through an angle $\Delta\phi$ about the vector $\vec{u}_z$. That is,

$$\overrightarrow{O_tb} = A \cdot \overrightarrow{O_ta} \tag{9}$$

where A is a rotation matrix. Here $\Delta\phi[\text{rad}] = w\Delta t$.

The change $\overrightarrow{ab}$ can be calculated as $\overrightarrow{ab} = \overrightarrow{O_tb} - \overrightarrow{O_ta}$. The new position $\vec{b}(b_x, b_y, b_z)$ in the laboratory frame can be calculated from the position $\vec{a}(a_x, a_y, a_z)$ as

$$\vec{b} = \vec{a} + \overrightarrow{ab}. \tag{10}$$

Position $\vec{c}(c_x, c_y, c_z)$, where the charged particle is located after moving from position $\vec{a}$ at time t to position $\vec{c}$ at time $t + \Delta t$, can be calculated by shifting an additional distance $V_{uz}\Delta t$ along the $\vec{u}_z$ direction from $\vec{b}$:

$$\vec{c} = \vec{b} + V_{uz}\Delta t\vec{u}_z. \tag{11}$$

The momentum $\vec{P}(P_x, P_y, P_z)$ at $\vec{a}$ also changes to a new value, $\vec{P}'$. The momentum of the particle at position $\vec{c}$ can be calculated using the same rotation matrix:

$$\vec{P}' = A \cdot \vec{P}. \tag{12}$$

For more details on how to reconstruct the trajectory of a charged particle, the Appendix presents our code for one step of the calculation. It is written in the ROOT platform [7]. Setting $l > 0$ in Eq. (9) produces the forward trajectory, while $l < 0$ yields the backward trajectory.

[1] Such as 0.1 cm, but this number can be adjusted according to the distance between available measurements and the homogeneity of the field.