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On-line statistical processing of radiation detector pulse trains with time-varying count rates

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ABSTRACT

Statistical analysis is of primary importance for the correct interpretation of nuclear measurements, due to the inherent random nature of radioactive decay processes. This paper discusses the application of statistical signal processing techniques to the random pulse trains generated by radiation detectors. The aims of the presented algorithms are: (i) continuous, on-line estimation of the underlying time-varying count rate $\theta(t)$ and its first-order derivative $d\theta/dt$; (ii) detection of abrupt changes in both of these quantities and estimation of their new value after the change point. Maximum-likelihood techniques, based on the Poisson probability distribution, are employed for the on-line estimation of θ and $d\theta/dt$. Detection of abrupt changes is achieved on the basis of the generalized likelihood ratio statistical test. The properties of the proposed algorithms are evaluated by extensive simulations and possible applications for on-line radiation monitoring are discussed.

1. Introduction

Due to the inherent random nature of radioactive decay, the output of any radiation detection system is subject to a significant amount of statistical fluctuation. In detectors operating in pulse mode, i.e., producing a short electrical pulse each time a particle is detected, measurement is essentially a counting process governed by the Poisson probability distribution. Proper statistical analysis is required in order to make correct estimates of the quantities derived from such measurements, as well as their expected precision [1]. This paper describes the application of modern statistical signal processing techniques [2] for the on-line analysis of radiation detector pulse trains.

The issues that will be dealt with are best exemplified by considering the operation of a radiation rate meter. This instrument provides a continuously updated reading of the average counting rate in a radiation detection system, in units of counts per unit time. A rate meter is required to fulfill two conflicting requirements: (a) the output should be stable, minimizing the inherent statistical fluctuations of nuclear counting, and (b) the instrument should quickly respond to an abrupt change in the radiation level. Requirement (a) is accomplished by increasing the averaging time constant. However, this has an adverse effect on requirement (b), since the instrument will delay in responding to a sudden rate change. With the advent of electronic microcontrollers, different processing algorithms have been developed for the realization of digital rate meters [3–6]. These algorithms try to achieve an optimum balance between output stability and response time. Their properties have been analyzed in a number of papers [7–9]. There have also been several attempts to develop self-adapting systems, with the aim to fulfill both requirements (a) and (b) simultaneously at a maximum level. They have been based either on statistical analysis [10] or on digital filtering techniques [9,11,12]. However, currently available high-level statistical methods have not yet been employed to this problem.

Another field of radiation measurement where statistical methods play a major role is radiation monitoring. Here, one is interested in detecting small changes in activity levels buried in the ambient background radiation. The detection sensitivity should be very high, but at the same time the rate of false alarms must be kept to a minimum. More sophisticated statistical techniques have been used in this direction. For example, the very sensitive *sequential probability ratio test* (SPRT) has been successfully employed in portal monitors [13–15]. Other examples may be found in Refs. [16–20].

The motivation for the current work started during an upgrade of the radioactive release monitoring system of the Greek Research Reactor, GRR-1. The amount of radioactive particulates

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released to the environment during reactor operation is measured according to the following procedure: air from the reactor exhaust stack is lead through a filter, where these particulates are trapped; the total particulate activity is continuously monitored by an adjacent detector. Due to the accumulative operation of the filter, the conversion of the detector output to the instantaneous activity release rate, in units of Bq per unit time, requires the estimation of the first-order time derivative of the counting rate. This proved to be a demanding task due to the large statistical fluctuations in the detector signal. The problems of numerical differentiation in the presence of noise are well documented [21]. Furthermore, the conversion to release rate has to be made continuously and fast, since a sudden increase in this quantity may indicate a fault in the reactor operation.

Although powerful mathematical methods to attack this problem may be found in the literature [2], they are mostly devoted to signals with Gaussian distributed noise. In this paper, algorithms suitable for processing the Poisson distributed output of radiation detectors are developed, with the following aims: (i) continuous on-line estimation of the counting rate and its first-order time derivative; (ii) quick detection of abrupt changes in both of these quantities. The approach has been inspired by recent advances in the fields of statistical process control (SPC), medical statistics and biology. Poisson counting statistics is also important in all of these fields. For example, in a manufacturing process one would count the number of faulty components produced during a certain operation time. Various SPC techniques have been developed for detecting changes in the failure rate [22–24] and for identifying linear time trends [25] or other kind of dependencies of the failure rate to external variables [26,27]. In disease surveillance one is interested for the sudden change in the rate of health incidents [28,29] and in biology the Poisson distribution is used to describe the temporal evolution of neuron firing rates [30,31].

2. Problem formulation, tools and algorithms

Let us consider a radiation detection system operating in pulsed mode. There are two distinct ways to analyze the random pulse train signal: either register the time between adjacent pulses or count the number of pulses in successive non-overlapping time intervals Δt. The latter method is technically easier to handle, since new data are produced at regular time intervals. If Δt is sufficiently small, little information is lost. This is the method adopted throughout the rest of this work.

Thus, the studied signal consists of a series of numbers $\{y_i\}$, $i = 1 \ldots k$, corresponding to the pulses registered during a time interval from $t_i - \Delta t$ to t_i. Each y_i is an independent random variable following the Poisson distribution

$$p_{\theta_i}(y_i) = \frac{\theta_i^{y_i} e^{-\theta_i}}{y_i!} \tag{1}$$

where θ_i is a real valued parameter. If the underlying count rate r is constant with time, then $\theta_i = \theta = r\Delta t$. In this case the Poisson distribution is called *homogeneous*. A time-dependent count rate $r(t)$ leads to the *inhomogeneous* Poisson distribution, which has the same form as Eq. (1) but with the parameter now given by $\theta_i = \int_{t_i-\Delta t}^{t_i} r(t)\,dt$. For simplicity, θ_i will be referred to as "count rate" in the rest of this paper, although it is actually a quantity without any physical units.

The sequence $\{y_i\}$ forms a discrete random signal with a joint probability or likelihood function given by

$$\Lambda(y_i; \theta_i) = \prod_{i=1}^{k} p_{\theta_i}(y_i). \tag{2}$$

Goal of the present work is the design of algorithms for processing the random signal $\{y_i\}$, in order to accomplish the following tasks:

(1) If θ is constant with time, estimate its value with the highest possible accuracy by utilizing a large number of measurements.
(2) If θ undergoes smooth variations with time, estimate its current value and its current rate of change.
(3) If θ exhibits abrupt changes, detect the onset of these changes and estimate the new value of θ after the change.

The proposed algorithms are intended for on-line application, operating in a *recursive* manner. This means that parameter estimates are based only on past measurements. The data are re-evaluated each time a new measurement becomes available and the estimated values are updated.

2.1. Counting rate estimation

Items (i) and (ii) involve the estimation of the underlying parameter θ from the data sequence $\{y_i\}$. The most widely used statistical methods of estimation employ the principle of maximum likelihood (ML). The estimated parameter $\hat{\theta}$ is found by maximizing the likelihood function, or, more conveniently, its logarithm. This may be formally written

$$\hat{\theta} = arg \sup_{\theta} \log \Lambda(y_i; \theta). \tag{3}$$

When θ is constant throughout the measurement window of width k, it is straightforward to show from Eq. (1)–(3) that

$$\hat{\theta} = \frac{1}{k}\sum_{i=1}^{k} y_i. \tag{4}$$

Thus, the ML estimator of θ in this case is just the sample average.

The estimation problem becomes more complex when θ varies with time. In parametric estimation [2], a model is assumed for the time dependence of θ, $\theta_i = g(t_i; \mathbf{a})$, where $g(t; \mathbf{a})$ is a model function depending on a set of parameters $\mathbf{a}$. Estimation of θ reduces to the estimation of these parameters. The quality of the estimation depends on how well the chosen model $g(t; \mathbf{a})$ describes the actual variation of θ, i.e., presupposes some *a priori* knowledge of the system behavior. To be able to follow possible system changes, the model parameters themselves may have to be considered as time dependent. This procedure has been extensively used in the engineering field of recursive system identification [32,33]. Very efficient computational methods have been devised for "tracking" the system parameters $\mathbf{a}$ when the observations y_i are of the form $y_i = g(t_i; \mathbf{a}) + \varepsilon_i$, where ε_i is a zero-mean normally distributed error term and the model function is linear in $\mathbf{a}$. These techniques are employed in a wide range of applications as, for example, adaptive signal processing, systems control, target tracking and navigation. Their parameter tracking ability is accomplished by exponentially weighting the data so that those around the current instant k have the highest significance for the estimation of $\mathbf{a}_k$. This is the idea behind the *recursive least squares* (RLS) algorithm with a "forgetting" factor $\lambda \in (0, 1)$ [32]. In RLS, $\mathbf{a}_k$ is estimated by minimizing the weighted sum of squares $\sum_{i=1}^{k} \lambda^{k-i} |y_i - g(t_i; \mathbf{a})|^2$.

However, these techniques are generally not well suited for counting data. The problems of applying least squares methods to Poisson distributed signals have been previously discussed [34,35]. Although the Poisson distribution may be approximated by a Gaussian at large values of θ, erroneous results, as for example negative count rates, may arise from least squares estimation in the low count rate region. Furthermore, the variance