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Last advances in analysis of intra-beam scattering in the hadron storage rings

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Abstract

Multiple intra-beam scattering (IBS) appears because of Coulomb scattering of charged particles on each other. In hadron storage rings, it results in re-distribution of the energy between different degrees of freedom, growth of six-dimensional emittance, development of non-Gaussian tails in distribution function and sometimes to the loss of particles. Traditional method of IBS analysis is based on "Gaussian model", which is assumed that distribution functions are Gaussian ones for all degrees of freedom. The basic theorems derived from the Gaussian model are shortly discussed. More detailed IBS analysis can be done using the Fokker–Planck equation (FPE) or equivalent tools. The FPE is written in "coordinate-momentum" space and in "invariant space" with account of the multiple IBS. Here, we consider two numerical methods of three-dimensional FPE solution developed at the last time: (1) the use of the "Langevin equations map" (LEM) and (2) the simulation of three-dimensional IBS using "the binary collision map" (BCM). The basics of both methods and examples of their application to storage rings are given. Alternative approach to the IBS analysis is named "molecular dynamics" (MD) based on the straight-forward integration of the particle trajectories taking into account the external electromagnetic fields and the Coulomb forces. Two modifications of MD are described: (1) a two-dimensional "wire model" and (2) a three-dimensional model of "periodical cells", which is fruitful in the case of small density beams, for example, the "crystalline beams". The basic problems of the molecular dynamics and results of its application to IBS analysis are shortly discussed.

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1. Introduction

In Terra Watt ACcumulator project (TWAC, ITEP, Moscow) ITEP storage ring for creation of the heavy ion beam with high energy level and high power of the extracted beam was proposed for use [1]. However, analysis of the accumulation process [2] has shown that one of the main problems is longitudinal heating of the beam due to multiple intra-beam scattering (IBS), which limits a power of the extracted beam. This effect has an important role in other modern accelerator facilities, which are characterized by high density of the stored beam in six-dimensional phase space. A goal of this report is to review the last advances in analysis of this effect.

As it is well-known, in infinite free space the multiple IBS tends to transform any initial distribution on momentums in Maxwellian distribution with equal temperatures on all degrees of freedom ("maxwellization" or "relaxation" process). A situation becomes more complicated if the space is finite (there are limits on maximal value of momentums); the diffusion flux on the border then results in loss of particle, which cool the gas. In hadron storage rings the beam is limited in momentum and coordinate space, and the equilibrium temperatures are different in different points of the magnetic lattice. Besides, in rings, the particles with different momentums have different deviations in space (this process is characterized by the

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so-named "dispersion function"). Both phenomena results in slow growth of the six-dimensional beam emittance. This growth is usually negligible for the accelerators where the particle life time is comparatively small; however, in the hadron storage rings the particle life time is large and the multiple IBS influences on the beam intensity and luminosity (in colliding beams).

A particle motion in circular rings can be considered as evolution of the particles invariants (longitudinal emittance and Courant–Snyder invariants). An evolution of average values of these invariants under action of multiple IBS (small angle Coulomb scattering) was considered in classical papers of Refs. [3–5]. The theory is based on application of "Gaussian model", which is assumed that the beam has Gaussian distribution in phase space on all three degrees of freedom. It is shown that an evolution of the Gaussian beam is described by a system of three differential equations for r.m.s. values of the beam invariants. Qualitatively, the IBS effects are defined by two fundamental theorems: (1) Piwinski's conservation law, derived for smooth focusing system and (2) Bjorken–Mtingwa theorem, which shows that, in strong focusing, the six-dimensional beam emittance is increased (except for a number of special points where its growth rate is equal to zero). The Gaussian models were used in numerous codes for the IBS simulations (e.g. see Refs. [6,7]).

Gaussian model often describes the beam behavior with good accuracy; however, sometimes (e.g., in presence of the beam losses or non-Gaussian tails of the distribution functions) we should use the general kinetic theory. Kinetic analysis of the multiple IBS is based on the solution of the Fokker–Planck equation (FPE) for the distribution function, which can be written in the momentum-coordinate space or in the "invariant space" where the FPE is an integro-differential equation since the friction and diffusion coefficients are many-dimensional integrals on the distribution function with very complicated kernel [8]. This equation can be simplified by the use of "Semi-Gaussian models", which allow us to reduce the FPE to one-dimensional (longitudinal) FPE [9,10]. The three dimensional FPE can be solved by two macroparticles methods: (1) the "binary collisions map" (BCM) that is used in the MOnte-CArlo Code (MOCAC) [11] and (2) the "Langevin equations map" (LEM) [12] that is based on the calculation of diffusion and friction coefficients (let us underline that this model is not self-consistent and can be used only for approximate estimates of the beam evolution).

The last method of the kinetic IBS analysis is named "molecular dynamics" (MD), which directly calculates the particle trajectories taking into account the Coulomb forces (here, the methods utilized for gravitational stellar systems are adapted to Coulomb systems). The main problem is a huge volume of computations; therefore, we are forced to use very simplified 2-D models [13] or to limit ourselves to a case of small intensity "crystalline" beams [14]. Nevertheless, this method has good prospects keeping in mind the future growth of the computer power.

2. Invariants of motion and its evolution

Let us introduce the vector $\vec{r}$ and the dimensionless momentum vector $\vec{P}$

$$\vec{r} = \begin{pmatrix} z - z_s \\ x \\ y \end{pmatrix}, \quad \vec{P} = \begin{pmatrix} \frac{1}{\gamma}\frac{\Delta p}{p} \\ x' \\ y' \end{pmatrix}. \tag{1}$$

Here z is the longitudinal particle coordinate, z_s is the longitudinal coordinate of the bunch center, and x and y are the horizontal and vertical transverse coordinates respectively, p is the particle momentum, Δp is its deviation from the equilibrium value, $x' = p_x/p$, $y' = p_y/p$ (p_x and p_y are the horizontal and vertical momentum components respectively). For linear motion of the particles in circular accelerators, the vectors $\vec{r}$ and $\vec{P}$ can be expressed through the action-angle variables J_m, and ψ_m, which can be considered as the components of "invariant-vector" $\vec{J}$ and "phase vector" $\vec{\psi}$ respectively. The invariants are defined by the following expression:

$$J_m = \gamma_m \tilde{r}_m^2 + 2\alpha_m \tilde{r}_m \tilde{P}_m + \beta_m \tilde{P}_m^2. \tag{2}$$

For a coasting beam (CB) $\alpha_1 = \gamma_1 = 0$, $\beta_1 = 1$; for a bunched beam (BB) $\gamma_1 = \nu_s^2/\gamma^2[(1/\gamma^2 - \alpha)R]^2$, where ν_s is the synchrotron tune, R is the average ring radius and α is the momentum compaction factor. For $m = 2, 3$ α_m, β_m, γ_m are "Twiss parameters" depending on longitudinal variable s. The vectors $\tilde{\vec{r}}, \tilde{\vec{P}}$ are defined by

$$\tilde{\vec{r}} = \begin{pmatrix} z - z_s \\ x - D\gamma P_1 \\ y \end{pmatrix}, \quad \tilde{\vec{P}} = \begin{pmatrix} \frac{1}{\gamma}\frac{\Delta p}{p} \\ x' - D'\gamma P_1 \\ y' \end{pmatrix}. \tag{3}$$

Here D, D' are the horizontal dispersion function and its derivative. The components of the vectors $\tilde{\vec{r}}, \tilde{\vec{P}}$ are expressed through invariants J_i and phases ψ_i by the formulae:

$$\begin{cases} \tilde{r}_i = \sqrt{J_i\beta_i}\cos\psi_i \\ \tilde{P}_i = -\dfrac{\alpha_i}{\beta_i}\tilde{r} - \sqrt{\dfrac{J_i}{\beta_i}}\sin\psi_i \end{cases}. \tag{4}$$

Kinematics of the IBS event (binary collision theory) is described in papers [3–5] where it is shown the Coulomb collision results in a change of the particle momentum $\vec{P}$. The invariant growth rates due to multiple IBS are connected with growth in momentum space, $\vec{P}$ space, using