Available online at www.sciencedirect.com
ScienceDirect
Nuclear Instruments and Methods in Physics Research A 577 (2007) 133–138
NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A
www.elsevier.com/locate/nima

Collective instabilities and collisional effects for space charge dominated beams

C. Benedetti*, G. Turchetti

Dipartimento di Fisica, Università di Bologna and INFN, Via Irnerio 46, 40126 Bologna, Italy

Available online 2 March 2007

Abstract

This paper is organized into two parts. In the first one we analyze the equipartitioning process in the presence of collisions for a Gaussian anisotropic beam. Far from instabilities the equipartition process can be described correctly by using Landau's collisional theory, whereas when the dynamics of the system is strongly nonlinear (e.g. in the vicinity of a resonance) a hybrid dynamical–collisional equipartition mechanism occurs. In the second part of the paper we discuss shortly a systematic study of the collective instabilities in a symmetric periodic focusing channel for a KV beam by using the moments method. We emphasize differences occurring when the periodic focusing lattice is replaced by the corresponding constant focusing one.

PACS: 29.27.Bd; 02.70.Ns; 41.75.–i; 52.59.Sa

Keywords: Beam dynamics; Collisional effects; Numerical simulations; Emittance growth

1. Introduction

Charged particle beams in drivers for heavy ion fusion are typically space charge dominated. As a consequence a careful investigation of the collective instabilities and resonances between the collective and nonlinear betatron oscillations is required in order to minimize the losses. Halo formation and growth is another relevant issue and its control is related to suppressing envelope mismatch and avoiding low-order resonances. Finally, in storage rings the collisional effects cannot be neglected because the storage time can be comparable with the relaxation time. For a coasting beam (or a beam with long bunches) we have developed a collisional two-dimensional model, which assumes an organization of the real particles into parallel charged filaments. A strong longitudinal coherence is implicitly assumed. The parameters of the model are the perveance and the bare phase advances of the confining lattice. For a beam with a longitudinal particle density $N_{\rm p}{\sim}10^{11}$ particles/m and a transversal dimension $R_{\rm beam}{\sim}1$ cm, the corresponding (physical) number of filaments is $N^{\rm (phys)}{\sim}10^6$ [1]. This model, taking into account Coulomb collisions, allows us to investigate the relaxation of the system towards thermodynamic equilibrium. In this context the relevant parameter is the thermodynamic relaxation time $\tau_{\rm relax}$, defined as the characteristic time required for the system to reach the equilibrium state specified by the self-consistent Maxwell–Boltzmann (MB) distribution. The analysis has been done in the constant focusing case. In the periodic focusing case it is unclear whether an MB equilibrium exists. In numerical simulations the number N of filaments that can be managed ranges from 10^3 to 10^4. Since usually N is smaller than $N^{\rm (phys)}$, a scaling law for the relaxation time is needed in order to infer the correct value of the relaxation time from the simulated data. In the limit $N \to \infty$ the charge distribution becomes continuous and the mean field theory is recovered. Starting from Landau's kinetic theory, we have obtained a formula for the scaling law towards the MB equilibrium; the law has been confirmed by numerical simulations. Once the calibration is made measuring the

*Corresponding author. Tel.: +39 051 2091123; fax: +39 051 247244.
E-mail address: benedetti@bo.infn.it (C. Benedetti).

0168-9002/$ - see front matter
doi:10.1016/j.nima.2007.02.087

relaxation time in a particular case (the calibration point), the scaling law provides the correct dependence of the collisional relaxation time from the system parameters (perveance, bare tunes, emittances and number of charged filaments). We introduce an equilibration time $\tau_{\rm EQ}$ defined as the characteristic time required for the horizontal and vertical temperatures of the system to equalize. In the presence of instabilities and in the vicinity of resonances, the equilibration is driven by collective dynamical processes rather than by collisions and usually $\tau_{\rm EQ} \ll \tau_{\rm relax}$. In this case the equilibration process is characterized by a time scale of $O(1)$ compared to the proper dynamic time of the system (dynamic equilibration process). On the other hand, far enough from this chaotic regime, collisions govern the equilibration process; $\tau_{\rm EQ}$ and $\tau_{\rm relax}$ appear to be strictly related and the equality $\tau_{\rm EP} \simeq \tau_{\rm relax}$ is supported by an analysis of the distribution and by the good agreement of the measured $\tau_{\rm EQ}$ with the scaling law obtained from Landau's kinetic theory (thermodynamic equilibration). In this paper we show that a third scenery is also possible. When a low-order resonance (e.g. the 2:2 Montague resonance) is approached, we observe an equilibration process which has a clear collisional origin, but substantial deviations from Landau's description are observed. We will show that this hybrid regime (dynamic–thermodynamic equilibration) is due to an interplay between collisions and the collective dynamics.

The collective instabilities are well known for a beam in a constant focusing channel [2]. In the case of a periodic focusing lattice, the analysis of the second order moments of the beam phase space distribution (envelope equation) shows the presence of unstable regions even in the case of a symmetric cell [3]. It is therefore relevant to investigate the stability properties of higher order collective modes. We have shown that the linear equations describing the evolution of the moments of a small perturbation on a KV beam can be written explicitly and, by applying Floquet theory, the stability condition can be worked out. We have found that for a beam in a symmetric FODO cell (ω_0 and ω being, respectively, the bare and the depressed phase advances) the sextupolar and octupolar perturbations are unstable in "tongue-like" regions of the parameter space $(\omega_0, \omega/\omega_0)$, similar to the ones where envelope parametric instabilities appear. Their union provides an instability domain where the PIC simulations show a significant emittance change. The region where the growth rate is large is similar to the region where emittance growth has been observed in experiments with a periodic lattice of identical electric quadrupoles [4].

2. The collisional model

We consider a coasting beam in a constant focusing channel and we assume that the charges are organized into N charged filaments parallel to the reference orbit. Denoting by $\mathbf{r}_i = (x_i, y_i)$ the transversal displacement of filament i and by $\mathbf{p}_i \equiv \mathrm{d}\mathbf{r}_i/\mathrm{d}s$ its conjugated momentum, the corresponding Hamiltonian is given by

$$H = \sum_{i=0}^{N} \left(\frac{p_{xi}^2 + p_{yi}^2}{2} + \omega_{x0}^2 \frac{x_i^2}{2} + \omega_{y0}^2 \frac{y_i^2}{2} \right) - \frac{\xi}{N} \sum_{i<j} \log(r_{ij}) \quad (1)$$

where r_{ij} is the distance between filaments i and j, ξ is the perveance, ω_{x0}/ω_{y0} are the bare phase advances (per unit length) and N is the number of filaments. The perveance does not depend on N and can be written (in the no-relativistic case) as $\xi = 2(q/m)(Q/v_0^2)$, where q and m are the charge and the mass per unit length of the filament whose ratio is equal to the ratio of the charge to mass of the single ion, and Q is the total charge per unit length in the beam. The Hamilton equations of motion related to Eq. (1) have been solved numerically on a parallel architecture by using a symplectic integrator and an optimal algorithm for the force field evaluation (the computational complexity was lowered from $O(N^2)$ to $O(N \log N)$) [1,5]. In numerical simulations it has been shown that an initial Vlasov stable distribution $\rho_0(\mathbf{x})$, where $\mathbf{x} = (x, y, p_x, p_y)$, relaxes to the MB distribution according to an exponential law

$$\rho(\mathbf{x}, s) \simeq \mathrm{e}^{-s/\tau_{\rm relax}} \rho_0(\mathbf{x}) + (1 - \mathrm{e}^{-s/\tau_{\rm relax}}) \rho_{\rm MB}(\mathbf{x}) \quad (2)$$

where the relaxation length $\tau_{\rm relax}$ increases linearly with N according to

$$\tau_{\rm relax} = NC\, f(\boldsymbol{\omega}_0, \boldsymbol{\varepsilon}, \xi) \quad (3)$$

and C is a calibration constant (which may depend on the type of the initial distribution ρ_0), $\boldsymbol{\omega}_0 = (\omega_{0x}, \omega_{0y})$ and $\boldsymbol{\varepsilon} = (\varepsilon_x, \varepsilon_y)$, where $\varepsilon_x, \varepsilon_y$ are the r.m.s. emittances. The form of the function f will be discussed later on in this paper. For an isotropic beam ($\omega_{0x} = \omega_{0y}$ and $\varepsilon_x = \varepsilon_y$), starting from an initially matched distribution, the second order moments remain constant and the relaxation process can be detected by looking at the distribution itself or at higher order moments of the distribution. For an anisotropic non-equipartitioned beam, the second order moments are no longer constant and can be used to analyze the relaxation process towards the MB equilibrium, provided that we are far from instabilities. In this case the (spatially averaged) instantaneous temperatures associated with the horizontal and the vertical degrees of freedom, defined as the incoherent part of the kinetic energy, are given by

$$k_{\rm B} T_x(s) \equiv \frac{\varepsilon_x^2}{\langle x^2 \rangle} = \langle p_x^2 \rangle - \frac{\langle x p_x \rangle^2}{\langle x^2 \rangle}. \quad (4)$$

A similar expression holds for $k_{\rm B} T_y$.

3. Kinetic theory

In the mean field approximation (neglecting collisions) the dynamics of the system is described by the Vlasov equation

$$\frac{\partial \rho}{\partial s} + [\rho, H] = 0, \quad \Delta V(x, y) = -4\pi \int \rho(\mathbf{x}, s)\, \mathrm{d}p_x\, \mathrm{d}p_y \quad (5)$$