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Nuclear Instruments and Methods in Physics Research A 579 (2007) 275–278
NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A
www.elsevier.com/locate/nima

Generation of customized wavelets for the analysis of γ-ray spectra

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Available online 24 April 2007

Abstract

The analysis of γ-ray spectra using wavelet analysis has recently been demonstrated as a new, promising technique for isotope identification with both NaI and CdZnTe spectra. Wavelet analysis has the potential to benefit over other approaches due to the fact that the signal can simultaneously be analyzed over multiple scales, thus eliminating potential false isotope identifications from artifacts such as the Compton edge and backscatter peaks. This implies that this algorithm is not impacted by variations in the detector's resolution, which changes with energy. We will present our results of optimizing this algorithm by custom designing wavelets for NaI spectra. Emphasis will be placed on quantifying the merits of these custom wavelets to traditional wavelets by exploring the quality of peak detection and localization, the first step to isotope identification.
Published by Elsevier B.V.

PACS: 29.30.Kv

Keywords: Wavelet analysis; Isotope identification; γ-ray spectroscopy

1. Introduction

Several recent studies have demonstrated that commercially available radiation isotope identifiers (RIIDs) have significant issues with correct isotope identification [1–3]. The majority of these devices, based on NaI detectors, utilize very simple identification algorithms, thus limiting their effectiveness in the fields of nuclear emergency response and homeland security. A new isotope identification algorithm for low-resolution spectra based on wavelet analysis has recently been described that is capable of accurate peak detection and localization [4,5]. This paper will demonstrate our recent progress with this algorithm with special mention of the creation of custom wavelets suited for the analysis of NaI spectra.

1.1. Analysis of γ-ray spectra with the continuous wavelet transform (CWT)

Wavelet analysis is the analysis of a signal simultaneously over both position and frequency (referred to as "scale" in the wavelet literature). This is a generalization of Fourier analysis where a signal is evaluated in frequency space only. Unlike Fourier analysis, wavelet analysis preserves the information of the locations of each frequency in the signal. As with Fourier analysis, this is achieved through a convolution-based transform over all γ-ray energies, E, and scales, s, given by

$$Wf(E,s) = T(E,s) = \int_{-\infty}^{\infty} f(t)\psi^*_{E,s}(t)\,\mathrm{d}t,$$

where W represents the wavelet transform of the signal $f(t)$, T is the wavelet transform, and ψ is the "mother" or "analyzing" wavelet. In the integral representation, t is a "dummy" variable of integration, which corresponds to the energy within our spectrum. As is obvious from the above equation, the result of the transform, $T(E,s)$, is a two-dimensional array of coefficients of both position (energy) and scale. These functions are typically presented as an image referred to as a scalogram. An example of a scalogram for a NaI ^{137}Cs spectrum is shown in Fig. 1.

There are many possible choices for the analyzing wavelet. Intuitively, the mathematical definition of a wavelet is a wave of finite duration whose net area is zero. Previous work has focused on the evaluation of several common wavelets for analyzing NaI spectra [5].

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0168-9002/$ - see front matter Published by Elsevier B.V.
doi:10.1016/j.nima.2007.04.055

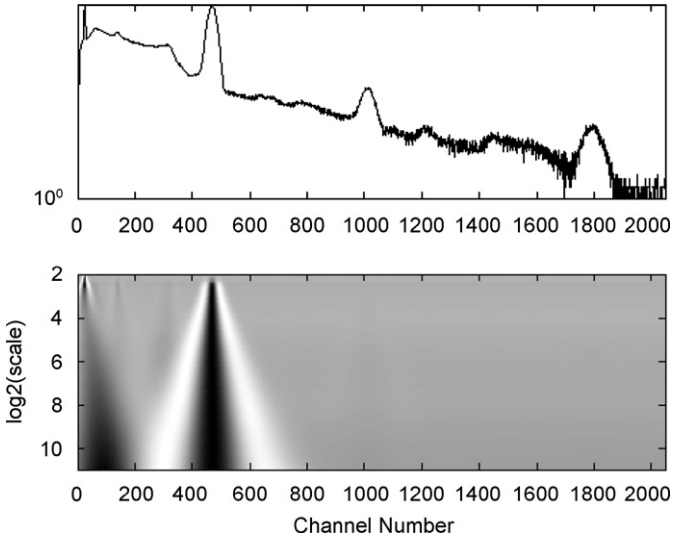

Fig. 1. Test signal (top) with sample scalogram (bottom). Dark shading indicates positions and scales of high correlation between the analyzing wavelet and the signal.

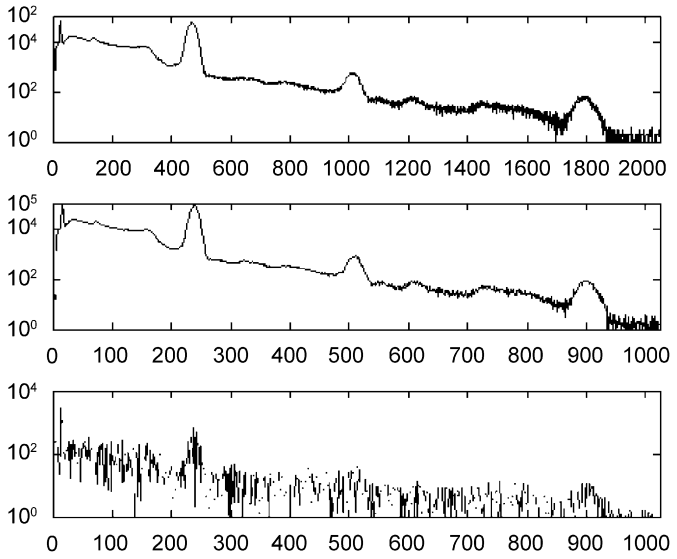

Fig. 2. Example of the discrete wavelet transform applied to a ^{137}Cs spectrum taken with NaI. The top shows the raw data, the middle plot shows the denoised approximation, and the bottom plot shows the noisy detail.

In order to analyze γ-ray spectra, it was previously demonstrated that analysis of the Lipschitz (or Holder) exponent provided an effective filter to locate peaks within a spectrum. The Lipschitz exponent is a measure of the differentiability of a signal. It can be shown that close to a discontinuity the signal can be approximated as

$$\left|T(E,s)\right| \leqslant As^{\alpha+\frac{1}{2}},$$

where A is an arbitrary constant, s is the scale, and α is the Lipschitz exponent [6]. For a perfect discontinuity (defined as a place where the first derivative is discontinuous), $\alpha = 0$. For an infinitely differentiable signal such as a Gaussian peak, α tends towards infinity.

Previous results demonstrated that peaks could be located in a γ-ray spectrum based on the Lipschitz exponent since they had larger values of α than other features in the spectrum such as electronics noise, Compton edges, and backscatter peaks [4,5]. A filter was established for features in the scalogram that spanned multiple scales such as the dark regions in Fig. 1 corresponding to features in the spectrum where only areas of the signal with sufficient α were considered relevant for the analysis (i.e. features identified by their Lipschitz exponent as a peak).

1.2. Denoising data with the discrete wavelet transform

It is important to note that, similar to the continuous and discrete Fourier transforms, there are two different implementations of the wavelet transform: the continuous and discrete transforms (CWT and DWT). The CWT uses every possible position and scale for the analyzing wavelet, which can be computationally intensive and generates significant amounts of data. In the DWT, the positions and scales are chosen by powers of two, resulting in faster analysis that can be just as accurate.

Through the DWT, the signal is split through a high- and low-pass wavelet filter into the details and approximations of the signal, respectively. This is shown for a γ-ray spectrum in Fig. 2. As shown, the low-pass filter has preserved many of the important features of the signal.

The DWT can be repeated several times and, through a thresholding technique, can be used as a signal-denoising algorithm. The general procedure is to first select the appropriate wavelets and number of iterations and compute the DWT. Next, a range of scales is chosen and threshold values for the coefficients in that range are selected. Finally, the inverse DWT is applied and the reconstructed, denoised spectrum is obtained.

2. Wavelet selection and custom wavelet generation

The previously reported results showed that three wavelets of the MATLAB Wavelet Toolkit demonstrated promise for locating peaks in NaI spectra: "gaus4," "coif2", and "bior2.6." These wavelets are shown in Fig. 3. That fact that these wavelets demonstrated the best results was slightly unexpected. A lower-order derivative, namely the second derivative of a Gaussian (all called the "Mexican Hat" wavelet or "mexh," also shown in Fig. 3), was expected to perform the best since it is largely the basis of many other, common peak-detecting algorithms [7]. However, this can be explained by considering that our signal is positive over its entire duration. Wavelets such as the Mexican Hat equally weight the positive and negative components of a signal. For a signal without negative components this is not ideal. Instead, wavelets such as "coif2" have more positive lobes than negative ones and