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Use of discrete Fourier transform to sum spectra in measurements with long counting times

M. Pilar Rubio Montero[a], Eduardo García-Toraño[b,*]

[a]*Departamento de Física, Universidad de Extremadura, Sta. Teresa de Jornet 38, E-06800, Mérida, Spain*
[b]*Laboratorio de Metrología de Radiaciones Ionizantes, CIEMAT, Avenida Complutense 22, 28040 Madrid, Spain*

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Abstract

When measuring nuclear spectra for long counting times, it is usual to split the acquisition in a set of spectra of shorter duration that are later added after correcting by drift during the measurements. We describe a method to determine the shifting correction by using the properties of the Discrete Fourier Transform (DFT). The method does not require assuming a particular spectral shape and the implementation is essentially independent of the kind of spectra being analyzed. The drift correction is defined for an integer number of channels. We present the results of the application to alpha-particle spectroscopy of long-lived nuclides.

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1. Introduction

The stability of measurement systems is essential to achieve the best spectral quality in nuclear measurements, particularly for those requiring weeks or months. Despite the improvements in instrumentation, shift and gain variations occur in the electronic chain. In many cases, the net effect is the apparent displacement of the spectra measured in a multichannel analyzer, at least in the region of interest, by a number of channels. In some cases, i.e. when counting rates are high, electronic stabilization systems can be of help. Usually, a peak in the spectrum is selected whose shape is maintained constant by a feedback mechanism at the level of the amplifier or analog-to-digital converter. Unfortunately, the procedure is useless when the counting rate is low and the use of a pulse generator to provide a reference for stability does not improve the situation when the main source of drift is the detector itself.

Another way of approaching the problem is to split the acquisition process in a series of measurements and to combine them after shifting each spectrum an appropriate number of channels so that all match a common pattern. A common practice is the determination of the maximum of an isolated peak in spectra by fitting, thus determining the adequate shifting correction before adding all spectra. The use of off-line gain stabilization algorithms based on the Stieltjes integral has been proposed recently [1]. We present here an alternative approach based on the use of the Discrete Fourier Transform (DFT) and describe its application to the measurement of alpha-particle spectra of long-lived nuclides, where counting times in the order of months can be necessary.

2. Theory

Inouye [2] was of the first to use Fourier-Transform-based techniques for the analysis of nuclear spectra. In his work about gamma-ray spectroscopy an analogy with communication theory is established, where the gamma-ray energy spectrum takes the place of a time function and the

*Corresponding author. Tel.: +34 91 346 6225; fax: +34 91 346 6442.
E-mail address: E.garciatorano@ciemat.es (E. García-Toraño).

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DFT is used to obtain the transformed function in the frequency domain. There, some mathematical operations such as smoothing and deconvolution can be carried out in an alternative (and sometimes easier) way to conventional numerical analysis. Examples of application to gamma-ray spectroscopy have been presented by Pinault [3], Madan et al. [4] and Hampton et al. [5] and to alpha-particle spectroscopy by Wätzig and Westmeier [6] and García-Toraño [7]. As a general rule, authors emphasize resolution enhancement and deconvolution in their studies, while in this paper we will focus on the problem of spectral shift.

A nuclear radiation spectrum can be mathematically represented by

$$\{Y_k\} k = 0, \ldots, N-1 \quad (1)$$

where N is the number of channels in the spectrum and Y_k is the contents of the kth channel. We suppose that a linear dependence exists between channel number and energy radiation. The DFT [8] of this spectrum is defined by

$$C_\omega = \sum_{k=0}^{N-1} Y_k \exp\left(-\mathrm{i}\frac{2\pi\omega k}{N}\right), \quad \omega = 0, \ldots, N-1 \quad (2)$$

where C_ω are complex coefficients and $\mathrm{i} = (-1)^{1/2}$. By analogy with the classical problem, ω will be called frequency of the transformation. Let us denote the real and imaginary parts of the C coefficients as $R(C)$ and $I(C)$, respectively. It is useful to define the amplitude and phase spectra as

$$|C_\omega| = \left|R^2(C_\omega) + I^2(C_\omega)\right|^{1/2} \quad (3)$$

and

$$\phi_\omega = \tan^{-1}\left(\frac{I(C_\omega)}{R(C_\omega)}\right). \quad (4)$$

In Fig. 1a we present a typical example of an alpha-particle spectrum of ^{244}Cm obtained with a Si detector. The corresponding amplitude $|C_w|$ and phase spectra ϕ_w are shown in Fig. 1b. As discussed in Refs. [2–7], the amplitude and phase spectra include two contributions: spectral information and noise. Both have different behavior in the frequency space: the information corresponding to the spectrum is essentially concentrated in the low-frequency region, while the noise contribution is uncorrelated channel to channel, and will approach a constant value, regardless of ω.

Let us now suppose that the measured spectrum is shifted by an integer number of channels m. By the properties of the DFT [8], the following relationship exists between the transforms of both (original and shifted) spectra:

$$Z_\omega = \frac{C_\omega}{Cs_\omega} = \left|\mathrm{e}^{-\mathrm{i}\omega m}\right| \quad (5)$$

where Cs_ω corresponds to the coefficients obtained in the transformation of the shifted spectrum. From Eq. (5) it can be seen that the amplitude spectrum of the function Z, obtained as the ratio of original and shifted spectra, will be

$$|Z_\omega| = 1$$

and the phase spectrum will be given by

$$\phi(Z_\omega) = \omega m. \quad (6)$$

Therefore, a fitting of ϕ as a function of ω will provide the value of m, the magnitude of the spectral shift. To compare two spectra and determine their relative displacement we will only need to compute the ratio of their DFT's and to calculate m from the phase spectrum.

Let us now shift the alpha spectrum shown in Fig. 1a by 2 channels. In Fig. 1c we present the amplitude and phase spectra of the corresponding Z function, calculated as indicated above. The plots show a good agreement with the theory: the amplitude spectrum $|Z(\omega)|$ is close to 1, and a linear fitting of the phase spectrum $\phi(Z(\omega))$ provides the right shifting factor. Since $Z(\omega)$ is obtained by dividing the transforms of two spectra, noise problems will be amplified at high frequencies. Therefore, as observed in the plots, information is more reliable in the low-energy frequency region. It is worth mentioning that the method neither requires the knowledge of the line shape, nor depends on it.

In a practical application of the method, the Z function will not be obtained as the ratio of two identical spectra. Let us suppose, for the sake of simplicity, that all spectra have equal counting times. Regardless of the drift effects, all spectra in a series of measurements will be slightly different due to the statistical fluctuations associated with the number of counts emitted and detected. We can expect that the information obtained from the frequency analyses will be deteriorated, as compared to that obtained from the example shown above.

3. Application to alpha-particle spectroscopy

One of the possible fields of application of the method is the measurement of alpha-particle spectra of long-lived nuclides. In order to have a good energy resolution, sources of weak activity must be used. A good example is the measurement of spectra to be used in the determination of alpha-particle emission probabilities (P_α). The requirements for the measurements are quite strict, so that sources of low activities must be used and, in order to reduce the fraction of coincidence-summing events, measurements must be carried out in a low solid angle, typically below 1%. Under these conditions, very low count rates are detected which lead to counting times in the order of months.

As a part of a recent experiment to measure P_α values of the nuclide ^{235}U [9], sources with activities of 2 Bq were measured at CIEMAT under solid angles $\Omega/4\pi \approx 0.006$. In order to acquire spectra with adequate statistics, counting times around 3 months were required. Therefore, it was decided to measure 2 series of 50 spectra each, with