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Entropy-based method to evaluate the data integrity

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Abstract

Projection stage of single photon emission computed tomography (SPECT) was discussed to analyze the characteristics of information transmission and evaluate the data integrity. Information is transferred from the source to the detector in the photon emitting process. In the projection stage, integrity of projection data can be assessed by the information entropy, which is the conditional entropy standing for the average uncertainty of the source object under the condition of projection data. Simulations were performed to study projection data of emission-computed tomography with a pinhole collimator. Several types of collimators were treated. Results demonstrate that the conditional entropy shows the data integrity, and indicate how the algorithms are matched or mismatched to the geometry. A new method for assessing data integrity is devised for those decision makers to help improve the quality of image reconstruction.

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Keywords: Data integrity; Entropy; Information theory; SPECT

1. Introduction

The SPECT-imaging process has two fundamental stages: projection and reconstruction. Comparison between information theory and emission-computed tomography (ECT) demonstrates the similarities: messages are encoded to signal first, and at the end of transmission the signal will be recovered to the messages in the information theory; the projection data of the source are detected, and reconstruction stage recover the source object in SPECT-imaging system. Information is transferred from the source to the detector in the photon-emitting process [1,2].

Similarity can be found between Shannon's information theory and emission-computed tomography, as shown in Fig. 1. The projection stage can be quantified rigorously by Shannon's information theory [3,4]. The emphasis of this paper is laid on projection process in ECT imaging as shown in Fig. 1.

In Section 2 we give some connotations of information theory, present some formulas, and apply the theory to SPECT imaging. The analysis in terms of collimator design is presented in Section 3. Here we obtain algorithm-independent measures of the data integrity and detection or recording capability of ECT and apply these measures to SPECT systems.

These all lead to a suggested philosophy for the assessment of the data integrity of pinhole collimator ECT imaging in the concluding sections.

2. Methodology

In Shannon's information theory we would like to describe uncertainty or perplexity as entropy. For two correlative variables, the conditional entropy demonstrates the average uncertainty of one variable on condition that the other is given, and the mutual information demonstrates the difference between both objects [3,4]. Physical connotation of some parameters in the information theory will be quantified in this part.

2.1. Definitions

We start with spatial trilinear orthogonal coordinates. The parameters with the footnote x are related to the object source or the input information, and those with y are

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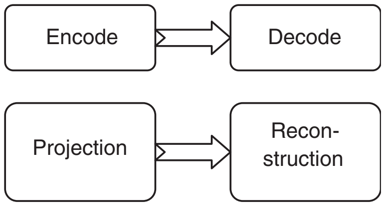

Fig. 1. Comparison between information theory and emission-computed tomography. The detection stage can be quantified rigorously using Shannon's information theory. The emphasis of this paper is laid on projection process in ECT imaging.

related to the projection data of the probe or the output information.

We consider a source voxel sequence $x_1, x_2, \ldots, x_m$, and $p(x_s)$ means the probability that a photon is emitted from the source voxel x_s. The projection data sequence $y_1, y_2, \ldots, y_n$ is also built, and y_k means a probe unit or a unit crystal of the detector, and $p(y_k)$ denotes the probability that a photon reaches the probe unit y_k. If quantities of photons are simulated, $p(y)$ describes the projection data of the probe while $p(x)$ images the object source.

Photons are emitted from the source to the detectors, and information is transferred in the emitting process. We now define the projection process as follows:

$$p\left(y_j\right) = \sum_i p\left(y_j/x_i\right)p(x_i) \tag{1}$$

where $p(y_j/x_i)$ means the probability that photons emitted from source voxel x_i can be caught by the probe unit y_j. Obviously $p(y_j/x_i)$ is determined by the system transition matrix.

2.2. Entropies and connotations

According to Shannon's information theory:

$$H(x) = -\sum_x p(x)\log p(x). \tag{2}$$

$H(x)$ is the entropy of the object source, which shows the amount of information the source contains, and denotes the uncertainty of the source x.

$$H(y) = -\sum_y p(y)\log p(y). \tag{3}$$

$H(y)$ is the entropy of the projection data, which shows the amount of the information that the detector unit y contains.

First we may just consider a single detector unit y_1:

$$H\left(x/y_1\right) = -\sum\sum p\left(x, y_1\right)\log p\left(x/y_1\right). \tag{4}$$

$H(x,\ y_1)$ is the entropy of the source under the given condition y_1, in other words $H(x,\ y_1)$ is the uncertainty of the source when the detector unit y_1 gets the projection data $g(y_1)$. With a view to the convenience for calculation and the mathematic relation between the conditional entropy $H(x,\ y_1)$ and the joint entropy $H(x,\ y_1)$, we use equation:

$$H\left(x/y_1\right) = H\left(x, y_1\right) - H\left(y_1\right). \tag{5}$$

In Eqs. (2)–(5), $p(x)$ may be initialized to $1/N$, where N is the number of voxels, because each voxel is the same as the other ones and the uncertainty is maximized in the case of uniform distribution, $p(y_1)$ and $p(x,\ y_1)$ can be quantified as follows:

$$p\left(y_1\right)(y_1) = \sum_x p(x)p\left(y_1/x\right) \tag{6}$$

$$p\left(x, y_1\right) = p(x)p\left(y_1/x\right). \tag{7}$$

2.3. Theory

Originally, the source contains information $H(x)$. Through the projection process we can take out part of the information, and the uncertainty of the source reduces gradually. At that time the amount of information the source contains is the conditional entropy $H(x,\ y_1)$.

Furthermore, we have been engaged thoroughly in research on all the detector units. When a new projection is carried out, we will pick up a new part of the information of the source. Fig. 2 presents the relation between the different entropies. Therefore the uncertainty of source will decrease throughout the projecting process, and the conditional entropy $H(x,\ y)$ will be diminished step by step.

The source will be quantified if

$$H(x/y) = 0 \tag{8}$$

which means the source has no uncertainty under the given condition of projections y.

2.4. Formulas

Mathematical calculations have been performed. From Fig. 2 we can see that the projection data of every detector unit contains information not only of the source but also of the previous detector units. We have to eliminate the mutual entropy between the units.

Considering the Bayesian formula:

$$p(x)p(y/x) = p(y)p(x/y) \tag{9}$$

and the relation between a new detector unit and previous units in the physical transmission:

$$\begin{aligned} p\left(y_1, y_2\right) &= p\left(y_1\right)p\left(y_2/y_1\right) \\ &= p\left(y_1\right)\sum_x\left[p\left(x/y_1\right)p\left(y_2/x\right)\right]. \end{aligned} \tag{10}$$

Formulas are educed:

$$\begin{aligned} H\left(x/y_1, y_2, \ldots, y_n\right) = H\left(x, y_1, y_2, \ldots, y_n\right) \\ - H\left(y_1, y_2, \ldots, y_n\right) \end{aligned}$$