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Treatment of non-Gaussian tails of multiple Coulomb scattering in track fitting with a Gaussian-sum filter

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Abstract

If any of the probability densities involved in track fitting deviate from the Gaussian assumption, it is plausible that a non-linear estimator which better takes the actual shape of the distribution into account can do better. One such non-linear estimator is the Gaussian-sum filter, which is adequate if the distributions under consideration can be approximated by Gaussian mixtures. The main purpose of this paper is to present a Gaussian-sum filter for track fitting, based on a two-component approximation of the distribution of angular deflections due to multiple scattering. In a simulation study within a linear track model the Gaussian-sum filter is shown to be a competitive alternative to the Kalman filter. Scenarios at various momenta and with various maximum number of components in the Gaussian-sum filter are considered. Particularly at low momenta the Gaussian-sum filter yields a better estimate of the uncertainties than the Kalman filter, and it is also slightly more precise than the latter.

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Keywords: Track fitting; Kalman filter; Gaussian-sum filter; Multiple Coulomb scattering

1. Introduction

The Kalman filter [1] has since many years been the default method for track finding and fitting in high-energy physics experiments. The popularity of the method is due to a number of attractive properties. In contrast to a global least-squares approach [2], no potentially large and non-diagonal covariance matrix of measurements needs to be inverted. Also, the existence of a Kalman smoother enables optimal estimates to be obtained anywhere along the track and not only at a reference surface.

Due to the recursive nature of the Kalman filter it can also be used for track finding [3]. At a given stage of the filter, several measurements in the next detector layer might be compatible with the prediction into that layer. The simplest strategy is to select the measurement closest to the predicted track and discard the rest. In an environment with large amounts of background and noise such a strategy will lead to too much inclusion of wrong measurements in the track candidates and subsequent loss of tracks. The currently most popular approach is therefore to split the track candidate into several branches, each branch corresponding to one of the compatible measurements in the detector layer [4]. When several layers with measurements are traversed, this procedure yields a combinatorial tree of track candidates. In the end, the best—according to e.g. the value of a χ^2-statistic—of the branches is kept.

The Kalman filter is known to be optimal when all probability densities involved during the reconstruction procedure are Gaussian. If some of these densities are non-Gaussian, it is plausible that a non-linear estimator which better takes the actual shape of the distribution into account can do better. One such estimator is the Gaussian-sum filter (GSF) [5], which is adequate when the distributions can be modelled by Gaussian mixtures. The state of the GSF becomes a Gaussian mixture, described by a set of parameter vectors, covariance matrices and weights. On the other hand, the state of the Kalman filter is described by a

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single parameter vector and the associated covariance matrix. The GSF resembles a set of Kalman filters running in parallel, each filter having a weight attached. At the end of the track fit procedure, the full information from the state distribution can be used by e.g. a dedicated GSF vertex fit [6]. If a single parameter vector and covariance matrix are desired, the first two moments of the mixture can be used.

The distribution of angular deflections due to multiple Coulomb scattering is known to have a Gaussian core but long, non-Gaussian tails. We present in this paper a GSF algorithm which uses a two-component Gaussian-mixture approximation of this distribution [7], with one component representing the core and the other component representing the tails. In the paper by Frühwirth and Regler a parameterization of the mixture variances and weights as a function of the radiation length of the scattering material is also available, enabling accurate calculations of the mixture parameters for a continuous range of the angle of incidence to the material layer.

2. Description of the algorithm

As the Kalman filter, the GSF proceeds by alternating prediction and update steps. Propagation of each of the components is done by a standard geometrical propagator, using the relevant track model. The parameter vector of each component is updated by a standard Kalman updator. The posterior weights are calculated in a non-linear way by taking the distances between the measurement and all predicted components into account. These calculations exhibit the adaptive behaviour of the GSF, as components close to the measurement tend to downweight components further away.

The state of the GSF at the entry of a layer of material in general consists of N components. Inclusion of multiple scattering effects in the layer amounts to a convolution of the probability densities, implying that the mixture after the convolution consists of $N \times M$ components, if the multiple scattering angle is described by an M-component mixture. An illustration of this effect is shown in Fig. 1. If many layers are traversed during the track fit, such a procedure can quickly lead to a combinatorial explosion in the number of components in the state. A strategy of restricting the number of components to a tolerable number with as little loss as possible of the information inherent in the mixture is therefore needed. The approach chosen in this work is to successively merge components being close in parameter space until a defined upper limit is reached. This procedure preserves the first two moments of the mixture.

3. Results from a simulation study

A simulation study comparing the performances of the GSF and a Kalman filter (KF) has been performed. Focusing primarily on qualitative, relative differences between the methods, a linear track model and a simple, planar detector geometry has been chosen. The detector geometry is shown in Fig. 2. There are 10 parallel detector planes of thickness 1% of a radiation length each in this geometry with 4 cm spacing between the planes. Each plane provides a measurement of the y-coordinate with uncertainty 30 μm. Tracks are simulated from the origin in a range of the angle of inclination β as indicated by the dashed lines in the figure and in a momentum range between 0.2 and 100 GeV/c. Multiple scattering in the detector layers has been simulated by drawing values of the scattering angle from a semi-Gaussian mixture [8]. This semi-Gaussian mixture has been shown to be a very good approximation of the true multiple scattering distribution. A global regression neglecting material effect is used as a preliminary fit, obtaining an initial estimate of the track parameters at the outermost layer. These are used as initial parameters for the KF and the GSF, which both run in the direction of the origin of the coordinate system. The reconstructed parameters (position and angle of inclination) are compared to the true parameters at the plane $x = 0$, forming the basis of evaluating the performance of the methods. For the KF, the estimated parameters are given directly by the algorithm. For the GSF, the mean value of the final state mixture is used. Only correct hits are used in the track fit, assuming a perfect outcome of the

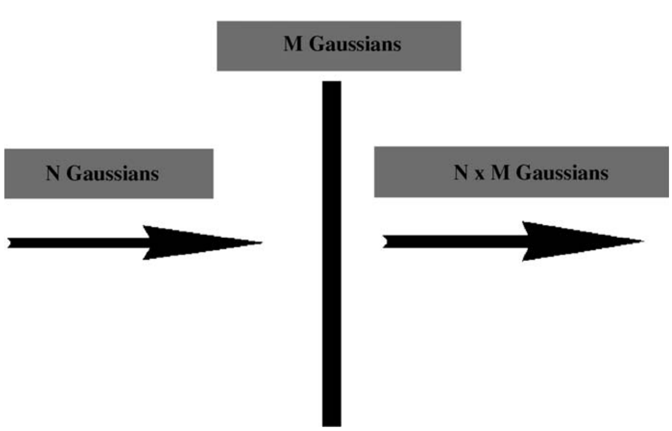

Fig. 1. Illustration of the effect of component multiplication.

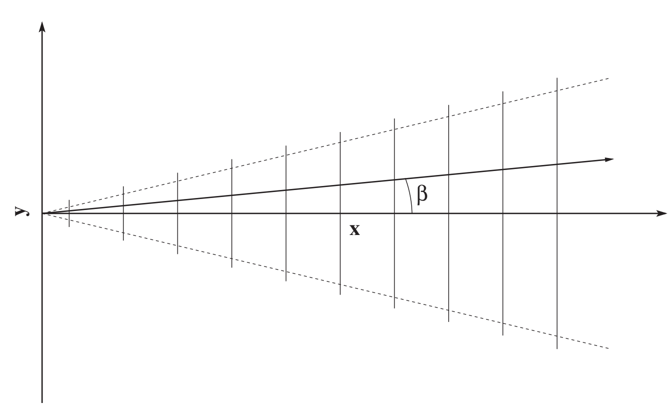

Fig. 2. A detector geometry consisting of 10 parallel detector planes.