

Neural network approach to parton distributions fitting

Andrea Piccione^{a,b,*}, Luigi Del Debbio^c, Stefano Forte^d, José I. Latorre^e, Joan Rojo^e,
The NNPDF Collaboration

^a*Dipartimento di Fisica Teorica, Università di Torino, Italy*

^b*INFN, Sezione di Torino, Via P. Giuria 1, I-10125, Italy*

^c*Theory Division, CERN, CH-1211 Genève 23, Switzerland*

^d*Dipartimento di Fisica, Università di Milano and INFN, Sezione di Milano, Via Celoria 16, I-20133, Italy*

^e*Departament d'Estructura i Constituents de la Matèria, Universitat de Barcelona, Diagonal 647, E-08028 Barcelona, Spain*

Available online 20 December 2005

Abstract

We will show an application of neural networks to extract informations on the structure of hadrons. A Monte Carlo over experimental data is performed to correctly reproduce data errors and correlations. A neural network is then trained on each Monte Carlo replica via a genetic algorithm. Results on the proton and deuteron structure functions and on the nonsinglet parton distribution will be shown.

© 2005 Elsevier B.V. All rights reserved.

PACS: 24.85.+p

Keywords: QCD; Parton distribution functions; Neural networks

1. Introduction

The requirements of precision physics at hadron colliders have recently led to a rapid improvement in the techniques for the determination of the structure of the nucleon. Playing this game factorization is a crucial issue. Indeed, it ensures that we can extract the parton structure of the nucleon from a process with only one initial proton (say, Deep Inelastic Scattering at HERA), and then we can use this as an input for a process where two initial protons are involved (Drell-Yan at LHC). In the QCD improved parton model the DIS structure function of the nucleon can be written as

$$F_2(x, Q^2) = x \left[\sum_{q=1}^{n_f} e_q^2 \mathcal{C}^q \otimes q_q(x, Q^2) + 2n_f \mathcal{C}^g \otimes g(x, Q^2) \right] \quad (1)$$

where $Q^2 = -q^2 = -(k - k')^2$, $x = Q^2/2p \cdot q$, and p , k and k' are the momenta of the initial nucleon, the incoming lepton, and the scattered lepton, respectively; \mathcal{C}^i are the coefficient functions perturbatively calculable, $q_q(x, Q^2)$ and $g(x, Q^2)$ the quarks and the gluon distributions that describe the nonperturbative dynamics, the so called Parton Distribution Functions (PDFs).

The extraction of a PDF from experimental data is not trivial, even if it is a well-established task. In order to do that we have to evolve the PDFs to the scale of data, perform the x -convolution, add theoretical uncertainties (resummation, nuclear corrections, higher twist, heavy quark thresholds, ...), and then deconvolute in order to have a function of x at a common scale Q^2 .

Recently, it has been pointed out that the uncertainty associated with a PDFs set is crucial [1–3]. The uncertainty on a PDF is given by the probability density $\mathcal{P}[f]$ in the space of functions $f(x)$, that is, the measure we use to perform the functional integral that gives us the expectation value

$$\langle \mathcal{F}[f(x)] \rangle = \int \mathcal{D}f \mathcal{F}[f(x)] \mathcal{P}[f(x)] \quad (2)$$

*Corresponding author. INFN, Sezione di Torino, Via P. Giuria 1, I-10125, Italy.

E-mail address: piccione@ge.infn.it (A. Piccione).

where $\mathcal{F}[f]$ is an arbitrary function of $f(x)$. Thus, when we extract a PDF we want to determine an infinite-dimensional object (a function) from finite set of data points, and this is a mathematically ill-posed problem.

The standard approach is to choose a simple functional form with enough free parameters ($g(x, Q_0^2) = x^\alpha(1-x)^\beta P(x)$), and to fit parameters by minimizing χ^2 . Some difficulties arise: errors and correlations of parameters require at least fully correlated analysis of data errors; error propagation to observables is difficult: many observables are nonlinear/nonlocal functional of parameters; theoretical bias due to choice of parametrization is difficult to assess (effects can be large if data are not precise or hardly compatible).

Here, we present an alternative approach to this problem. First, we will show our technique applied to the determination of the Structure Functions. This is the easiest case, since no evolution is required, but only data fitting, thus it is a good application to test the technique. Then, we will show how this approach can be extended for the determination of the PDFs.

2. Structure functions

The strategy presented in Refs. [4,5] to address the problem of parametrizing deep inelastic structure functions $F(x, Q^2)$ is a combination of two techniques: a Monte Carlo sampling of the experimental data and a neural network training on each data replica.

The Monte Carlo sampling of experimental data is performed generating N_{rep} replicas of the original N_{dat} experimental data,

$$F_i^{\text{(art)}(k)} = (1 + r_N^{(k)} \sigma_N) \left[F_i^{\text{(exp)}} + r_i^{s,(k)} \sigma_i^{\text{stat}} + \sum_{l=1}^{N_{\text{sys}}} r_i^{l,(k)} \sigma_i^{\text{sys},l} \right] \quad (3)$$

where $i = 1, \dots, N_{\text{dat}}$, r are Gaussian random numbers with the same correlation as the respective uncertainties, and $\sigma^{\text{stat}}, \sigma^{\text{sys}}, \sigma_N$ are the statistical, systematic and normalization errors. The number of replicas N_{rep} has to be large enough so that the replica sample reproduces central values, errors and correlations of the experimental data.

The second step is to train a neural network on each data replica. A neural network [6] is a highly nonlinear mapping between input and output patterns as a function of its parameters. We choose an architecture with four inputs ($x, \log x, Q^2, \log Q^2$), two hidden layers with five and three neurons, respectively, and one output, $F(x, Q^2)$. The training on each replica is performed in two steps. First, we use the Back Propagation technique to minimize

$$\chi_{\text{diag}}^{2(k)} = \frac{1}{N_{\text{dat}}} \sum_{i=1}^{N_{\text{dat}}} \frac{(F_i^{\text{(art)}(k)} - F_i^{\text{(net)}(k)})^2}{(\sigma_i^{\text{stat}})^2} \quad (4)$$

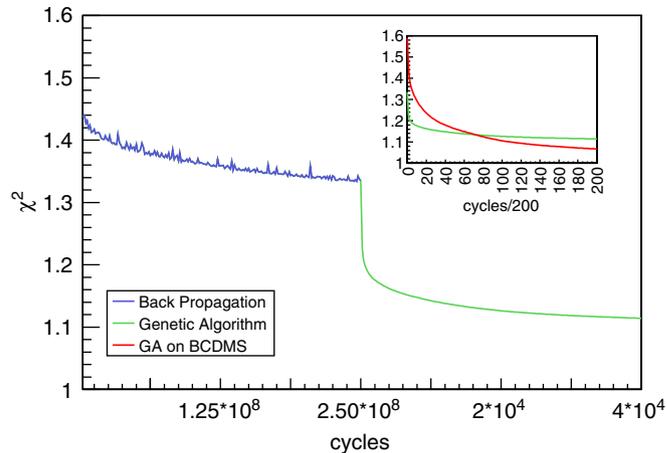


Fig. 1. Dependence of the χ^2 on the length of training: (big pad) total training (small pad), detail of the GA training.

then, we use the Genetic Algorithm [7] to minimize

$$\chi^{2(k)} = \frac{1}{N_{\text{dat}}} \sum_{i,j=1}^{N_{\text{dat}}} (F_i^{\text{(art)}(k)} - F_i^{\text{(net)}(k)}) \times \text{cov}_{ij}^{-1} (F_j^{\text{(art)}(k)} - F_j^{\text{(net)}(k)}). \quad (5)$$

The Back Propagation technique allows for a fast minimization, but it always oscillates, while the Genetic Algorithm is always decreasing, and it is more suitable for the last part of the training where the stability of the χ^2 is needed, see Fig. 1. Once we have trained all the neural networks on data replicas, we have a probability density in the space of structure functions, $\mathcal{P}[F(x, Q^2)]$, which contains all information from experimental data, including correlations. Expectation values over this probability measure are then evaluated as averages over the trained network sample,

$$\langle \mathcal{F}[F(x, Q^2)] \rangle = \frac{\sum_{k=1}^{N_{\text{rep}}} \mathcal{F}(F^{\text{(net)}(k)}(x, Q^2))}{N_{\text{rep}}}. \quad (6)$$

In Fig. 2 we show our results¹ for the deuteron structure function $F_2^d(x, Q^2)$ [4], and for the proton structure function $F_2^p(x, Q^2)$ [5] compared to a polynomial parametrization [8]. We observe that in the data range the two fits agree within errors. In the extrapolation region the error band of the polynomial fit has the same narrow width as in the data range, while the error band of the neural networks grows indicating that we are in a region where the error is underdetermined since there are no data.

Neural networks turn to be a suitable tool also in the presence of incompatible data. Indeed, once a good fit is obtained, say a stable value of $\chi^2 \sim 1$, the neural networks infer a natural law by following the regularity of data, and incompatible data are discarded without any hypothesis on the shape of the parametrization (see Fig. 3).

¹The source code, driver program and graphical web interface for our structure function fits is available at <http://sophia.ecm.ub.es/f2neural>.

Download English Version:

<https://daneshyari.com/en/article/1833040>

Download Persian Version:

<https://daneshyari.com/article/1833040>

[Daneshyari.com](https://daneshyari.com)