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Precise heavy-quark masses

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Abstract

We report on determinations of the running masses for charm- and bottom-quarks from deep-inelastic scattering reactions and for the top-quark from hadro-production of top-quark pairs. The running masses in the $\overline{\text{MS}}$ scheme can be extracted with good precision at next-to-next-to-leading order in QCD. In the global fits to data the full correlations of the extracted mass parameters with the parton distributions in the proton and with the strong coupling constant α_s are kept. For charm- and bottom-quarks the method provides complementary information on these fundamental parameters from hadronic processes with space-like kinematics. The measured top-quark mass is confronted with the Monte Carlo top-quark mass parameter determined from a comparison to events with top-quark decay products. The Monte Carlo mass is not identical with the pole mass. Its translation to the pole mass scheme introduces an additional uncertainty of the order of 1 GeV.

Keywords: heavy-quark hadro-production, heavy-quark structure function in deep-inelastic scattering, charm-quark mass, bottom-quark mass, top-quark mass, running masses in $\overline{\text{MS}}$ scheme

1. Introduction

Quark masses are fundamental parameters of the gauge theory of the strong interactions, Quantum Chromodynamics (QCD). They are, however, not directly observable due to confinement. Quark masses appear in the theory predictions for cross sections or other measurable quantities and, as such, they are subject to the definition of a renormalization scheme once quantum corrections at higher orders are included. In many QCD applications the pole mass is the conventional scheme choice. The heavy-quark's pole mass m_q^{pole} is introduced in a gauge invariant and well-defined way at each finite order of perturbation theory as the location of the single pole in the two-point correlation function. The pole mass scheme is, in fact, inspired by the definition of the electron mass in Quantum Electrodynamics. For heavy quarks, however, this has its short-comings [1, 2], because due to confinement quarks do not appear as free particles in asymptotic states in the S-matrix. Therefore, the pole mass m_q^{pole} must acquire non-perturbative corrections, because in the full theory the quark two-point function does not display any pole. This leads to an intrinsic uncertainty in the definition of m_q^{pole} of the order of Λ_{QCD} related to the renormalon ambiguity [3].

Fortunately, one can consider alternative definitions based on the (modified) minimal subtraction in the $\overline{\text{MS}}$ scheme, which realizes the concept of a running mass $m_q(\mu)$ at a scale μ of the hard scattering in complete analogy to the treatment of the running strong coupling $\alpha_s(\mu)$. As a benefit, predictions for hard scattering cross sections in terms of the $\overline{\text{MS}}$ mass display better convergence properties and greater perturbative stability at higher orders.

More generally, one can define so-called short-distance masses $m_q(R,\mu)$, where R is a scale associated with the scheme. The $\overline{\text{MS}}$ mass is then just one example of a short-distance mass $m_q(R,\mu)$ with R taken at the scale $R \sim m_q$. Other schemes define a so-called

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$1S$ mass [4, 5] through the perturbative contribution to the mass of a hypothetical ${}^3S_1$ toponium bound state or a "potential-subtracted" (PS) mass [6]. As alternative renormalization schemes, all short-distance masses $m_q(R,\mu)$ can be related to the pole mass $m_q^{\rm pole}$ through a perturbative series,

$$m_q^{\rm pole} = m_q^{\rm MSR}(R,\mu) + \delta m_q(R,\mu)\,, \qquad (1)$$

$$\delta m_q(R,\mu) = R \sum_{n=1}^{\infty} \sum_{k=0}^{n} a_{nk}\, \alpha_s^n(\mu)\, \ln^k\left(\frac{\mu^2}{R^2}\right)\,, \qquad (2)$$

with coefficients a_{nk} known to three loops in QCD [7, 8].

As quark masses are not physical observables the determination of m_q relies on the comparison of a theory prediction $\sigma_{\rm th}(m_q)$ for a cross section with the experimentally measured value $\sigma_{\rm exp}$ for a given observable and kinematics as the best fit solution to the equation $\sigma_{\rm exp} = \sigma_{\rm th}(m_q)$. The accuracy of this approach is intrinsically limited by the sensitivity $\mathcal{S}$ of $\sigma_{\rm th}(m_q)$ to m_q,

$$\left|\frac{\Delta\sigma}{\sigma}\right| = \mathcal{S} \times \left|\frac{\Delta m_q}{m_q}\right|\,. \qquad (3)$$

Thus, for a given experimental error or a theoretical uncertainty $\Delta\sigma$ on the cross section, the greater the sensitivity $\mathcal{S}$ the better the accuracy for m_q can be achieved.

In this review we discuss several determinations of the running masses in the $\overline{\rm MS}$ scheme for charm- and bottom-quarks from deep-inelastic scattering (DIS) reactions and for the top-quark from hadro-production of top-quark pairs, which have been performed within the Collaborative Research Center/Transregio 9 (CRC/TR 9). We briefly describe the theoretical prerequisites and the global fits to data for the extraction of the running masses at next-to-next-to-leading order (NNLO) in QCD. We stress, that is is important to keep the full correlations of the extracted mass parameters with other non-perturbative parameters entering the cross sections predictions, such as the parton distribution functions (PDFs) in the proton and the reference value of the strong coupling constant $\alpha_s(M_Z)$.

The results reported here are quoted in the *2014 Review of Particle Physics* [9] of the particle data group (PDG) and the presentation in this article follows previous reports [10, 11] on the subject with updates.

2. Charm-quark mass

Cross sections for the production of heavy-quarks in DIS are particularly well suited to confront the quark mass dependence of theoretical predictions in perturbative QCD with experimental measurements in space-like kinematics. For the production of charm-quarks in neutral (NC) or charged current (CC) DIS there exists very precise data from the HERA collider and from fixed-target experiments.

In QCD the DIS heavy-quark structure functions F_k which parametrize the hadronic cross section are subject to the standard factorization

$$F_k(x,Q^2,m_q^2) = \sum_{i=q,\bar{q},g} \left[f_i(\mu^2) \otimes C_{k,i}\left(Q^2, m_q^2, \alpha_s(\mu^2)\right)\right](x)\,, \qquad (4)$$

where $k = 1,2,3$. Q^2 and x are the usual DIS kinematical variables and m_q is the heavy-quark (pole) mass. The perturbative coefficient functions $C_{k,i}$ are known for CC to next-to-leading order (NLO) [12, 13] and at asymptotic values of $Q^2 \gg m_q^2$ even to NNLO [14, 15, 16]. For NC the coefficient functions have been computed approximately to NNLO [17, 18, 19, 20].

In eq. (4) we also display all dependence on the other non-perturbative parameters, i.e. the PDFs f_i for light quarks and gluons as well as the strong coupling constant α_s. The conversion to the running mass definition follows the standard procedure for changing the renormalization condition, i.e. $m_q \to m_q(\mu)$ at the respective order in perturbation theory. This has been discussed in refs. [21, 22] and the specific implementation for DIS heavy-quark production in eq. (4) has been presented in refs. [23, 24].

The parametric dependence of the DIS structure functions F_k in eq. (4) on m_q can be used for a determination of the heavy-quark mass. The sensitivity of this procedure relates directly to the corresponding uncertainty on the measurements of F_k, see eq. (3). For charm production in NC DIS the nucleon structure function F_2 displays a sensitivity $\mathcal{S} \sim 1.5$ which implies that an experimental accuracy of 4% for F_2 translates into an uncertainty of 3% for the charm-quark mass [23]. With the precision of current DIS data for charm production this suggests an error on $m_c(m_c)$ of O(few)% as the ultimate precision in the approach based on inclusive structure functions.

Starting from eq. (4) we have extracted the $\overline{\rm MS}$ charm-quark mass $m_c(m_c)$ in several phenomenological studies [25, 26] (and variants [24, 27, 28]) based on world data for deep-inelastic scattering and fixed-target data as well as data from the Large Hadron Collider (LHC) for the Drell-Yan process The use of the running mass in global analyses in the fixed-flavor number scheme (FFNS) (with $n_f = 3$) is twofold. It allows for a comparision of the extracted mass parameter to the world average published by the PDG as a consistency check. In addition, due to better convergence and