Available online at www.sciencedirect.com

ScienceDirect

Nuclear and Particle Physics Proceedings 261–262 (2015) 140–154

www.elsevier.com/locate/nppp

Massive Feynman integrals and electroweak corrections

Janusz Gluza[a], Tord Riemann[b,*]

[a]*Institute of Physics, University of Silesia, Uniwersytecka 4, 40007 Katowice, Poland*
[b]*15711 Königs Wusterhausen, Germany*

Abstract

We describe selected advances in the calculation of electroweak corrections to massive scattering problems at colliders, from the very beginning in the nineteen seventies until contemporary developments. Recent years brought a remarkable progress due to new calculational technologies. This was motivated by demands from phenomenological applications at particle accelerators: higher multiplicities of the final states, extreme kinematics, need of higher precision and thus of higher orders in perturbation theory.

Keywords: Quantum field theory, perturbation theory, Feynman integrals, collider physics

1. Introduction

The calculation of observable quantities for high energy colliders became more and more involved in recent years, although the basic understanding of perturbative quantum field theory has been settled decades ago. The term "calculation" has two sides here to be taken into account, of quite different origin. First, one has to derive formulae for the quantity of interest, with a sufficient accuracy in order to match experimental needs. This is part of theoretical work in the classical understanding. But, by time the answers get more involved, both in quantity and in complexity. Also, the singularity behaviour becomes worse. As a consequence, the result of theoretical research to be disseminated is often not only an analytical formula written in an article, but also a piece of more or less sophisticated software. This is fine, but it raises new questions of cooperation. Software has to be supported in a rapidly developing world of computing. How to distribute software in an appropriate manner, thereby respecting the authors' rights in a satisfactory way, but at the same time not too much hindering its use? Let us remind that software use in nearly all realistic cases means also adaptation and so changing the original creation.

Since we are working in the field of particle phenomenology since decades, we collected some experience with all these aspects, to some extent we even contributed to the culture of practicioning. We come back to the point in section 7.

In sections 2 to 6 we survey part of research performed in the research group B1 "Massive particle production" of Sonderforschungsbereich/Transregio 9 of Deutsche Forschungsgemeinschaft. Due to the calculational difficulties it took few years after the concise formulation of the perturbative renormalization of the electroweak theory by t' Hooft and Veltman [1, 2] and after the invention of SCHOONSCHIP [3]. A famous piece of work was Veltman's study of the ρ parameter with the observation that high particle masses may show up at low energy [4]. First detailed studies of the calculational techniques and of the consequences for phenomenology came out soon, notably [5, 6]. Since then, much effort has been concentrated to the refinement of predictions of perturbative effects in the Standard Model and beyond.

Calculations have been done for many quantities, notably the weak corrections to the Z boson parameters ρ_Z and $\sin^{2,eff}_W$; at one loop e.g. in [7, 8, 9, 10], and later

---

[*] corresponding author
Email address: `tordriemann@gmail.com` (Tord Riemann)

http://dx.doi.org/10.1016/j.nuclphysbps.2015.03.012

also with higher order corrections predicted by the electroweak theory and by QCD [11, 12, 13, 14, 15]. These higher-order calculations have to be performed, but they have also to be inserted into phenomenological tools.

Although a lot of the material presented here is applied also to LHC physics, we will concentrate on higher-order contributions to e^+e^- annihilation, mainly arising from loop corrections:

$$e^+e^- \to f^+f^-,\ \ f^+f^-(\gamma),\ \ f^+f^-\gamma,\ \ f^+f^-\gamma(\gamma). \quad (1)$$

A large part of the present study is devoted to the treatment of single Feynman integrals. They are the building blocks of Feynman diagrams related to some observable. We will consider an arbitrary L–loop integral $G(X)$ with loop momenta k_l , with E external legs with momenta p_e and with N internal lines with masses m_i and propagators $1/D_i$:

$$G(X) = \frac{1}{(i\pi^{d/2})^L} \int \frac{d^dk_1 \dots d^dk_L\, X(k_1, \dots, k_L)}{D_1^{n_1} \dots D_i^{n_i} \dots D_N^{n_N}}, \quad (2)$$

with

$$\begin{aligned} d &= 4 - 2\epsilon, & (3)\\ D_i &= q_i^2 - m_i^2 = \left[\sum_{l=1}^{L} c_i^l k_l + \sum_{e=1}^{M} d_i^e p_e\right] - m_i^2, & (4)\end{aligned}$$

where $X(k_1, \dots, k_L)$ stands for tensors in the loop momenta.

2. ZFITTER

ZFITTER [16, 17] is a long-term project, dating back to the nineteen seventies. The aim is a state-of-the-art description of

$$e^+e^- \to (\gamma, Z) \to f^+f^-(n\gamma) \quad (5)$$

in the Standard Model. A description of the project has been published quite recently [18]. Since 1989 ZFITTER is among the standard software packages for the description of the Z boson resonance at LEP. Further, it was used for predictions of the top-quark and Higgs-boson masses from radiative corrections in the Standard Model prior to their discoveries. Until about 1992, ZFITTER rested mainly on theoretical work done by its authors on complete one-loop electroweak corrections in the Standard Model. In the nineteen nineties it become more and more important to integrate higher-order corrections derived by other authors and to support the users from experimental groups, notably from DELPHI, L3, OPAL, and also from the LEPEWWG. This is documented in the "LEP electroweak working group report" of 1995 [11] and references therein. The seminal review studies of (5) by the LEP community for LEP 1 in 2005 [19] and LEP 2 in 2013 [20] rest to a large extent on ZFITTER v.6.42 [16, 17].

ZFITTER became the "etalon" software for the Z-boson resonance studied for many years at LEP 1 und at LEP 2. Among the main results of LEP are the following, quoted from the "Review of Particle Physics" (2012) [21]:

$$\begin{aligned} M_Z &= 91.1876 \pm 0.0021\ \text{GeV},\\ \Gamma_Z &= 2.4952 \pm 0.0023\ \text{GeV},\\ \sin^2\theta_{\text{weak}} &= 0.22296 \pm 0.00028,\\ \sin^2\theta^{\text{eff}}_{\text{lept}} &= 0.23146 \pm 0.00012,\\ \sin^2\theta^{\text{MS}}_{Z} &= 0.23116 \pm 0.00012,\\ N_\nu &= 2.989 \pm 0.007. \quad (6)\end{aligned}$$

A similar analysis, also based on ZFITTER, has been published by ALEPH, DELPHI, L3, OPAL, LEP-EWWG in 2013 [20]. The constraint

$$m_t = 178^{+11}_{-8}\ \text{GeV} \quad (7)$$

is obtained from the virtual corrections, in good agreement with the much more precise direct measurement of about $m_t = 173.2 \pm 1$ GeV [22]. For the Higgs boson mass, they predict:

$$\begin{aligned} M_H &= 118^{+203}_{-64}\ \text{GeV},\ \ \text{only LEP},\\ M_H &= 122^{+59}_{-41}\ \text{GeV},\ \ \text{plus}\ m_t,\\ M_H &= 148^{+237}_{-81}\ \text{GeV},\ \ \text{plus}\ M_W,\ \Gamma_W,\\ M_H &= 94^{+29}_{-24}\ \text{GeV},\ \ \text{plus}\ m_t,\ M_W,\ \Gamma_W. \quad (8)\end{aligned}$$

An update is [23], where it is quoted $M_H = 89^{+22}_{-18}$ GeV, or $M_H < 127$ GeV (90% c.l.). In 2012, the LHC collaborations discovered a scalar particle with a mass of about 125 GeV [24, 25]. The present best value is $M_H = 125.6 \pm 0.3$ GeV [26].

This might be illustrated by the famous blue band plot of the LEPEWWG [27, 28], which we reproduce in figure 2, together with the presumably first proposal of an electroweak precision plot in figure 1. The development of precision predictions is nicely illustrated in figures 3 to 5.

It is pointed out in [20] that there are, besides ZFITTER, two alternative approaches for precision Standard Model tests available. One approach is practiced in the "Review of particle physics" of the Particle Data Group [29], which traces to a large extent back to ZFITTER. The second approach is the Gfitter project. In fact, at the webpage http://gfitter.desy.de/ one finds a lot of data similar to the blue band plot