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Non-perturbative computation of the strong coupling constant on the lattice

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Abstract

We review the long term project of the ALPHA collaboration to compute in QCD the running coupling constant and quark masses at high energy scales in terms of low energy hadronic quantities. The adapted techniques required to numerically carry out the required multiscale non-perturbative calculation with our special emphasis on the control of systematic errors are summarized. The complete results in the two dynamical flavor approximation are reviewed and an outlook is given on the ongoing three flavor extension of the programme with improved target precision.

Keywords: QCD, running coupling, quark mass, lattice QCD, Monte Carlo, Schrödinger functional, gradient flow

1. Introduction

Quantum Chromo Dynamics (QCD) is the renormalizable quantum field theory containing gluon and quark fields that interact in a unique way dictated by SU(3) gauge invariance. It may be seen as arising from the standard model of elementary particles in a limit where all other fields, including their interactions with quarks and gluons, are stripped away. Strong interactions and confinement are the characteristics of this sector which hence calls for non-perturbative evaluations and is in the focus of lattice formulations and simulations.

We here consider QCD with a free number of N_f color triplets (flavors) of quark species. In Nature we see the case $N_\mathrm{f} = 6$ with the flavors up, down, strange, charm, bottom and top in order of ascending mass. The species beyond light up and down quarks come with characteristic scales of the order of 0.1 GeV, 1 GeV, 4 GeV, 175 GeV. Therefore it makes sense to consider effective theories with $N_\mathrm{f} < 6$ to describe physics with characteristic energies significantly below the scales of the dropped degrees of freedom. They then enter only indirectly into the determination of the free parameters of the effective theory. In lattice simulations the modelling of the precise flavor content is technically very demanding. Therefore a lot of studies are found and will also be discussed here that refer to $N_\mathrm{f} = 2$ and $N_\mathrm{f} = 3$ where the latter number is the minimum to allow for real applications as an effective theory [1, 2, 3, 4, 5]. In any case generalized QCD has $N_\mathrm{f} + 1$ free parameters given by one quark mass per species and in addition the gauge coupling.

The two light species are in most studies, including those described here, approximated to be degenerate. The value zero for some or even all N_f quark masses is theoretically nice as it enhances the chiral symmetry of the model and is thus stabilized under renormalization. The renormalization of the coupling can be defined in this massless limit and we then speak of a massless renormalization scheme. Such schemes are technically convenient in nontrivial perturbative as well as non-perturbative calculations. The renormalization of the coupling can be left unchanged as quark masses are 'turned on later'. To define a renormalized coupling constant in a massless scheme an additional scale μ enters via the renormalization conditions. The resulting scale dependent 'running' coupling $\bar{g}(\mu)$ obeys a Callan-

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Symanzik equation

$$\mu \frac{d}{d\mu} \bar{g}(\mu) = \beta(\bar{g}(\mu)), \quad (1)$$

in which the function β is determined by the theory once a particular coupling definition has been adopted. A negative β-function corresponds to asymptotic freedom. The free integration constant that arises in solving this differential equation can be taken as the free parameter that the bare coupling has been 'traded for' in the process of renormalization. It may be fixed by specifying $\bar{g}$ for a specific μ value in GeV. Alternatively one may convert the Callan-Symanzik equation into the equivalent integral statement that

$$\begin{aligned} \Lambda &= \mu \left(b_0 \bar{g}^2(\mu)\right)^{-b_1/2b_0^2} \exp[-1/(2b_0\bar{g}^2(\mu))] \quad (2) \\ &\times \exp\left[-\int_0^{\bar{g}(\mu)} \left\{\frac{1}{\beta(x)} + \frac{1}{b_0 x^3} - \frac{b_1}{b_0^2 x}\right\} dx\right] \end{aligned}$$

is independent of μ. In this equation b_0, b_1 are the leading and scheme independent coefficients in the asymptotic expansion

$$\beta(x) = -\sum_{n\geq 0} b_n x^{2n+3}, \quad (3)$$

$$b_0 = \frac{1}{(4\pi)^2}\left(11 - \frac{2}{3}N_f\right), \quad (4)$$

$$b_1 = \frac{1}{(4\pi)^4}\left(102 - \frac{38}{3}N_f\right). \quad (5)$$

For asymptotically large μ it is sufficient to evaluate (2) with the perturbative series for β truncated beyond some $n \geq 1$. Therefore, in a *perturbative* context, Λ is associated with the behavior of $\bar{g}(\mu)$ for $\mu \to \infty$.

2. Hadronic renormalization scheme and finite size scaling

In lattice simulations also non-perturbative quantities associated with scales of order one GeV and below can be computed in principle. Examples are the masses of light hadrons and matrix elements involving their one-particle states, decay constants like f_π, f_K for example [6, 7]. This opens up the possibility to also match such quantities directly to experiment and in this way determine the free parameters of QCD which can then be determined with an in principle arbitrary precision[1]. This is not true if perturbation theory at any finite energy is involved, since with an asymptotic expansion – even if very high orders were available – an uncertainty remains. This effect is expected to be small at the Z-mass, but the situation is much more delicate for example for determinations of α_s in the τ-mass region.

As a conceptually simple example of a hadronic scheme one could imagine to use as input parameters the mass of the proton and in addition the masses of N_f types of stable mesons that are sensitive to the respective quark masses. In practice one of course has $N_f + 1$ dimensionless parameters at ones disposal in the lattice theory of which N_f may be determined by dialing the correct ratios of meson to proton mass. The remaining degree of freedom allows to tune the lattice theory to its critical point where the continuum limit is reached. Due to asymptotic freedom in QCD this is accomplished by sending the bare coupling to zero. In this limit, all dimensionfull quantities emerge in the form of well-defined multiples of appropriate powers of the proton mass which we thus employ to set the scale for all observables. Equivalently we may say that all that is computed from theories including the lattice and compared with experiment are dimensionless ratios of observables. The above scheme selects a minimal set of independent mass ratios and, with these tuned, all other ratios must 'fall in place'. We try to be very explicit on this seemingly trivial issue, as sometimes confusion seems to arise here nevertheless.

In the previous paragraphs we have described a rather idealized situation. For various technical reasons we will not use this precise hadronic scheme, and in addition several sources of in practice unavoidable systematic errors have to be taken into account in lattice computations.

A lattice that is simulated on a computer necessarily has a finite number of sites and thus finitely many degrees of freedom. This implies a finite extent L and a finite spacing or resolution a such that one has $(L/a)^4$ sites. In large present day simulations $L/a \sim 100$ is achieved. If we refer to $m_{\rm had}$ as some hadronic mass scale, then $am_{\rm had} > 0$ represents a distortion of the physics by an unphysical UV cutoff effect. Details depend on the chosen lattice discretization, but in practice and employing Symanzik's theory of cutoff effects [8, 9, 10], we expect these effects to diminish asymptotically at a rate proportional to $(am_{\rm had})^2$. We need to verify that we have reached this asymptotic behavior to estimate the prefactor by multiple simulations in which the resolution (and nothing else) is varied. This whole procedure is called continuum extrapolation and, of course, leaves behind a contribution in the final error

[1] This refers to pure QCD. Other interactions are still neglected.