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Resonance Chiral Lagrangians and alternative approaches to hadronic tau decays

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Abstract

Exclusive semi-leptonic decays of the tau lepton offer a clean probe to study the hadronization of QCD currents in its non-perturbative regime and learn about resonance dynamics, which drives strong interactions in these processes. In this theory outlook, I will use the simplest non-trivial di-pion tau decays to illustrate briefly recent theoretical progress on these analyses and their comparison to data.

Keywords: Resonance Chiral Lagrangians, Hadronic tau decays

1. Introduction

We will focus here on exclusive hadronic decays of the tau lepton. An updated detailed account on this topic, containing inclusive analyses, leptonic tau decays, and CPV and LFV searches in tau decays as well, can be found in Ref. [1].

The matrix element for $\tau^- \to H^- \nu_\tau$ decays, where H stands for the final-state hadrons, can be written

$$\mathcal{M}(\tau^- \to H^- \nu_\tau) = \frac{G_F}{\sqrt{2}} V_{ud/us}\, \bar{u}_{\nu_\tau} \gamma^\mu (1-\gamma_5) u_\tau \mathcal{H}_\mu \,, \quad (1)$$

in which

$$\mathcal{H}_\mu = \left\langle H \middle| (V-A)_\mu e^{i\mathcal{L}_{QCD}} \middle| 0 \right\rangle = \sum_i (\ldots)^i_\mu F_i(q^2, \ldots) \quad (2)$$

is the hadronic matrix element of the left-handed QCD current evaluated between the initial hadronic vacuum and the final-state mesons. In eq. (2), $(\ldots)^i_\mu$ are the set of allowed Lorentz structures and $F_i(q^2,\ldots)$ the hadronic form factors, scalar functions depending on the kinematical invariants $(q^2,\ldots)$, which reduce to the charged-meson decay constants (F_π, F_K) for one-meson tau decays. This, in turn, are well-known from the measured $(\pi/K)^- \to \mu^- \bar{\nu}_\mu$ decay rates [2]. Multi-meson modes start to provide non-trivial information on the hadronization of QCD currents.

A model/theory is needed to compute the $F_i(q^2,\ldots)$ in this case, and observables are readily obtained either directly in terms of them, or using their appropriate combinations, the structure functions [3].

$M_\tau \sim 1.8$ GeV implies that the dynamics of hadronic tau decays will be mostly influenced by the lowest-lying light-flavored resonances like $\rho(770)$ or $a_1(1260)$, so that their propagation must be accounted for in the hadronic form factors.

Among the many approaches that have been developed with this purpose, let us mention the Gounaris-Sakurai (GS) [4] and Kühn-Santamaría (KS) [5] parameterizations and the Resonance Chiral Lagrangians ($R\chi L$) approaches [6] that we will be discussing.

These input form factors are fitted to data, directly or by means of a Monte Carlo Generator (which also helps to better estimate the backgrounds for a given process), being TAUOLA [7, 8, 9] the standard one in these low-energy applications. Related developments [10, 11, 12] are of interest both for theorists and experimentalists, for flavor-factories and colliders.

This kind of analyses should render the determination of resonance parameters: masses, widths and couplings. Contrary to the most common practice, model parameters should be avoided for the first two. Instead, phys-

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ically meaningful model independent parameters shall be used, as they are those defined by the pole position of the resonance in the complex plane

$$\sqrt{s^{pole}_{Res}} = M^{pole}_{Res} - \frac{i}{2}\Gamma^{pole}_{Res}\,, \quad (3)$$

which should be the ones quoted by the PDG [2].

ALEPH, CLEO, DELPHI and OPAL first [13, 14, 15, 16, 17, 18, 19, 20], and then the two-flavor factories -BaBar and Belle- [21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34] have been providing data on exclusive hadronic decay modes of the tau lepton with increasing precision and the prospects for Belle-II [35] and future planned facilities [36, 37] are very much promising. This demands a corresponding effort on the theoretical description of these decays. Recent theoretical developments on hadronic tau decays are discussed in the next section using the well-known two-pion vector form factor (VFF) to explain them.

2. An illustrative example: The two-pion VFF

The KS-like parameterizations of the hadronic form factors are built fixing their normalizations to the leading order Chiral Perturbation Theory result [38], as linear weighted combinations of Breit-Wigner factors accounting for the dominant exchanged resonances. For instance, in the case of the $\pi^-\pi^0$ vector form factor

$$\left\langle \pi^-(p)\,\pi^0(p')\middle|\bar{d}\,\gamma_\mu\,u\middle|0\right\rangle = \sqrt{2}\,(p-p')_\mu\,F_V^{\pi^-\pi^0}(s), \quad (4)$$

where $s = (p + p')^2$, the KS-parameterization reads

$$F_V^{\pi^-\pi^0}(s) = \frac{BW_\rho(s) + \alpha\,BW_{\rho'}(s) + \beta\,BW_{\rho''}(s)}{1 + \alpha + \beta}\,, \quad (5)$$

with

$$BW_R^{KS}(s) = \frac{M_R^2}{M_R^2 - s - i\,\sqrt{s}\,\Gamma_R(s)}\,, \quad (6)$$

where the off-shell resonance width, $\Gamma_R(s)$, is obtained from the absorptive part of loop functions involving pions and Kaons. The GS expressions add to eq.(6) a contribution resembling the one produced by the real part of these loops, shifting both numerator and denominator.

However, both KS and GS parameterizations do violate the low-energy expansion of QCD [39, 40] at next-to-leading order [41], introducing a systematic error in the analyses using them [1]. This bias can (and should) be avoided by approaches built on the basis of chiral symmetry, as they are the $R\chi L$. In fact, these intend to interpolate between the two known extreme regimes of QCD: the chiral limit at low energies and the operator product expansion (OPE) of QCD at high energies ($E > M_\tau$). An extensive program [6, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52] has been developed to work out the restrictions imposed on the resonance couplings by the OPE. The existence of a consistent minimal set of short-distance constraints applying to the two- and three-point Green functions and related form factors in the resonance region has been shown in both intrinsic parity sectors [53, 54]. These were obtained in the $N_C \to \infty$ limit [55, 56, 57] within the single resonance approximation including multi-linear operators in resonance fields and working the latter in the antisymmetric tensor formalism. Such procedure provides a sound theoretical basis for the $R\chi L$ and their application to study two- and three-meson tau decays.

A complementary approach uses dispersion relations to obtain the hadronic form factors. In this way, analyticity and unitarity are automatically fulfilled and the poorest known (high-E) region is suppressed by the subtractions of the dispersive integrals. Consequently, results are also less sensitive to the precise short-distance QCD constraints, minimizing the effect of the error associated to the $1/N_C$ expansion.

Again, in the case of the charged two-pion VFF one would have [58]

$$F_V^{\pi^-\pi^0}(s) = \exp\left[\alpha_1\,s + \frac{\alpha_2}{2}\,s^2 + \frac{s^3}{\pi}\int_{s_{\rm thr}}^{\infty} ds'\,\frac{\delta_1^1(s')}{(s')^3(s'-s-i\epsilon)}\right], \quad (7)$$

for the three-subtractions [2] case, where the phaseshift, $\delta_1^1(s)$ is obtained as

$$\tan\delta_1^1(s) = \frac{\Im m\left[F_V^{\pi^-\pi^0(0)}(s)\right]}{\Re e\left[F_V^{\pi^-\pi^0(0)}(s)\right]}\,, \quad (8)$$

using an input form factor $F_V^{\pi^-\pi^0(0)}(s)$, which can be provided by the $R\chi L$ [59, 60, 61, 62, 63]

$$F_V^{\pi^-\pi^0\,(0)}(s) = \frac{M_\rho^2}{M_\rho^2\left[1 + \frac{s}{96\pi^2F_\pi^2}\left(A_\pi(s) + \frac{1}{2}A_K(s)\right)\right] - s} \quad (9)$$

$$= \frac{M_\rho^2}{M_\rho^2\left[1 + \frac{s}{96\pi^2F_\pi^2}\Re e\left(A_\pi(s) + \frac{1}{2}A_K(s)\right)\right] - s - iM_\rho\Gamma_\rho(s)}\,.$$

[1] Analyticity and UV QCD constraints are also violated [42].

[2] These are α_1 and α_2 -which are taken as free parameters- and $F_V^{\pi^-\pi^0}(0) = 1$, which is fixed by CVC.