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The anomalous magnetic moment of the muon: Theory update

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Abstract

We present a review on recent progress in perturbative calculations for the anomalous magnetic moment of the muon. We present recent calculations for leptonic contributions to $g-2$ and discuss the NNLO contributions to hadronic vacuum polarisation insertions.

Keywords: QED, $g-2$, hadronic contributions

1. Introduction

The anomalous magnetic moment of the muon $(g-2)_\mu$ has been both experimentally measured and theoretically calculated with astonishing precision. The difference between the experimental value [1, 2]

$$a_\mu^{\rm exp} = 0.001\,165\,920\,80(54)(33)[63] \qquad (1)$$

and the theory prediction [3]

$$a_\mu^{\rm theo} = 0.001\,165\,918\,40(59) \qquad (2)$$

has the size of about three standard deviations. On the theory side the contributions to $a_\mu^{\rm theo}$ can be decomposed into three parts

$$a_\mu^{\rm theo} = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm hadr}\,, \qquad (3)$$

where $a_\mu^{\rm QED}$, $a_\mu^{\rm EW}$, and $a_\mu^{\rm hadr}$ denote the QED, electroweak, and hadronic contributions, respectively. The error on the theory prediction (2) is dominated by the uncertainty of the hadronic contributions [4, 5].

The electro-weak contributions have been calculated analytically in Refs. [6, 7, 8, 9, 10] and with the measurement of the mass of the Higgs boson all input parameters are now sufficiently well known which leads to the following contributions up to next-to-leading order

$$\begin{aligned}
a_\mu^{EW,(1)} &= (194.80 \pm 0.01)\times 10^{-11},\\
a_{\mu,{\rm bos}}^{EW,(2)} &= -(19.97 \pm 0.03)\times 10^{-11},\\
a_{\mu,f_{\rm rest},H}^{EW,(2)} &= -(1.50 \pm 0.01)\times 10^{-11},\\
a_\mu^{EW,(2)}(e,\mu,u,c,d,s) &= -(6.91 \pm 0.20 \pm 0.30)\times 10^{-11},\\
a_\mu^{EW,(2)}(\tau,t,b) &= -(8.21 \pm 0.10)\times 10^{-11},\\
a_{\mu,f_{\rm rest},noH}^{EW,(2)} &= -(4.64 \pm 0.10)\times 10^{-11},\\
a_\mu^{EW,\geq 3\ell} &= (0 \pm 0.20)\times 10^{-11}\,.
\end{aligned}$$

The full result for the electro-weak corrections reads

$$a_\mu^{EW} = (153.6 \pm 1.0)\times 10^{-11}$$

with a conservative error estimate.

The QED contributions have been calculated up to five-loop order in [3, 11]. We want to stress that looking at the absolute size of the QED corrections (cf Tab. 1) one finds that the four-loop contribution is of the same size as the difference between theory and experiment. Therefore it is mandatory to verify the only existing calculation of these contributions by an independent one. First steps towards this are presented in Section 2. Corrections to $a_\mu^{\rm QED}$ from vacuum polarization insertions have been calculated up to five-loop order and are discussed in Section 4. Even though the corrections contained in $a_\mu^{\rm hadr}$ are of non-perturbative nature they still

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receive quantum corrections which can be addressed in perturbation theory and are discussed in Section 3.

2. Leptonic contributions at four-loop order

The pure QED contributions can be further decomposed as

$$a_\mu^{\rm QED} = \sum_{n=1} \left(\frac{\alpha}{\pi}\right)^n A_\mu^{(n)} \qquad (4)$$

$$A_\mu^{(n)} = A_1^{(n)} + A_2^{(n)}(M_e/M_\mu) + A_2^{(n)}(M_\mu/M_\tau) + A_3^{(n)}(M_e/M_\mu, M_\mu/M_\tau) \qquad (5)$$

$$A_2^{(4)}(M_e/M_\mu) = n_l^3 A_2^{(43)} + n_l^2 A_2^{(42)a} + n_l^2 n_h A_2^{(42)b} + \cdots \qquad (6)$$

where $A_1^{(n)}$ contains the universal contribution and n_l and n_h denote light electron and heavy muon loops, respectively.

In Ref. [13] a first step towards an independent calculation of the electronic contributions $A_2^{(4)}(M_e/M_\mu)$ to the anomalous moment of the muon has been made. Contributions with at least two closed electron loops have been calculated. The results are accurate up to terms M_e/M_μ and are shown in the following

$$A_2^{(43)} = \frac{1}{54}L_{\mu e}^3 - \frac{25}{108}L_{\mu e}^2 + \left(\frac{317}{324} + \frac{\pi^2}{27}\right)L_{\mu e} - \frac{2\zeta_3}{9} - \frac{25\pi^2}{162} - \frac{8609}{5832} \approx 7.196\,66\,, \qquad (7)$$

$$A_2^{(42)a} = L_{\mu e}^2\left[\pi^2\left(\frac{5}{36} - \frac{a_1}{6}\right) + \frac{\zeta_3}{4} - \frac{13}{24}\right] + L_{\mu e}\left[-\frac{a_1^4}{9} + \pi^2\left(-\frac{2a_1^2}{9} + \frac{5a_1}{3} - \frac{79}{54}\right) - \frac{8a_4}{3} - 3\zeta_3 + \frac{11\pi^4}{216} + \frac{23}{6}\right] - \frac{2a_1^5}{45} + \frac{5a_1^4}{9} + \pi^2\left(-\frac{4a_1^3}{27} + \frac{10a_1^2}{9} - \frac{235a_1}{54} - \frac{\zeta_3}{8} + \frac{595}{162}\right) + \pi^4\left(-\frac{31a_1}{540} - \frac{403}{3240}\right) + \frac{40a_4}{3} + \frac{16a_5}{3} - \frac{37\zeta_5}{6} + \frac{11167\zeta_3}{1152} - \frac{6833}{864} \approx -3.624\,27\,,$$

$$A_2^{(42)b} = \left(\frac{119}{108} - \frac{\pi^2}{9}\right)L_{\mu e}^2 + \left(\frac{\pi^2}{27} - \frac{61}{162}\right)L_{\mu e} - \frac{4\pi^4}{45} + \frac{13\pi^2}{27} + \frac{7627}{1944} \approx 0.494\,05\,,$$

loop order	with α^{-1}(Rb)[$\times 10^{-11}$]	with $\alpha^{-1}(a_e)$[$\times 10^{-11}$]
1	116 140 973.318 (77)	116 140 973.212 (30)
2	413 217.6291 (90)	413 217.6284 (89)
3	30 141.902 48 (41)	30 141.902 39 (40)
4	381.008 (19)	381.008 (19)
5	5.0938 (70)	5.0938 (70)
a_μ(QED)	116 584 718.951 (80)	116 584 718.845 (37)

Table 1: QED corrections by loop order using values for α obtain from the anomalous magnetic moment of the electron [11] and the second best determination of α from Ref. [12]. The errors are indicated originate from α, the parametric uncertainty of the mass ratio M_e/M_μ and the error from the numerical integration.

group	$10^2 \cdot A_{2,\mu}^{(4)}(M_\mu/M_\tau)$	
	Ref. [14]	Ref. [3]
I(a)	0.00324281(2)	0.0032(0)
I(b) + I(c) + II(b) + II(c)	−0.6292808(6)	−0.6293(1)
I(d)	0.0367796(4)	0.0368(0)
III	4.5208986(6)	4.504(14)
II(a) + IV(d)	−2.316756(5)	−2.3197(37)
IV(a)	3.851967(3)	3.8513(11)
IV(b)	0.612661(5)	0.6106(31)
IV(c)	−1.83010(1)	−1.823(11)

Table 2: Results from Ref. [14] in comparison with Ref. [3]. We refer to Ref. [3] for the definition of the diagram classes.

with $L_{\mu e} = \ln(M_\mu^2/M_e^2)$, $\zeta_n = \sum_{k=1} 1/k^n$, $a_1 = \ln 2$ and $a_n = {\rm Li}_n(1/2)$, $n \geq 4$. Excellent agreement with the results in the literature has been found.

The contributions from τ-leptons to the anomalous magnetic moment of the muon can very efficiently be calculated by performing an asymptotic expansion in the mass ratio $z = M_\mu/M_\tau \approx 6 \cdot 10^{-2}$ leading to a power series

$$A_2^{(4)}(M_\mu/M_\tau) = \sum_{n=1}^{\infty} C_{4,n} z^{2n}\,. \qquad (8)$$

After performing the expansion on the diagram level one is left with at most the calculation of four-loop vacuum diagrams, which have been extensively studied in