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Nuclear and Particle Physics Proceedings 267–269 (2015) 175–179

www.elsevier.com/locate/nppp

Strings from large charged fields

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Abstract

It has been known for a long time that the low energy dynamics of open-strings is described by non-abelian gauge theories in the same way that the low energy effective description of closed strings is governed by Einstein's gravity. In this short note, we review and comment on some examples where conversely, an effective behavior of non-abelian gauge theories in some particular limits are described by some type of extended objects like strings or branes. We constrain the discussion to a few examples sharing some similarities.

Keywords: Gauge Theory, String Theory, AdS/CFT Correspondence,

1. Introduction

This short note is based on a talk given in the X Latin American Symposium of High Energy Physics held in Medellin-Colombia from 24th to 28th of November of 2014, and was intended to address a non-string theory target audience. The main aim is to collect a few known scenarios where the physics of a particular set of operators in a field theory, can be effectively be described by string-like or even brane-like configurations at some particular limits.

Stringy behaviors embedded in field theory are not a new topic. A classical example corresponds to vortex in $D = 4$ Abelian-Higgs model. There we can consider static topological solutions confined on two spatial dimension Abrikosov [1], Nielsen and Olesen [17], known as Abrikosov-Nielsen-Olesen vortex, which in three spatial dimensions look like a rigid string. Those solutions can also be found in supersymmetric extensions where they saturate the BPS condition of the supersymmetric algebra and hence can be protected from quantum corrections, allowing access to the strong coupling regime of the supersymmetric theory. It is also possible to find an effective action describing the dynamics of those strings.

A related example is given by flux tubes between a quark-antiquark pair in QCD. Lately, there has been a renewed interest on QCD-strings (see for example Aharony and Komargodski [2], Shifman and Yung [20]) mainly motivated for recent improvements measuring the energies of various states of such strings via lattice simulations (see Teper [21] for a review and references therein).

Another interesting embedding of strings into gauge theory observables concern to the scattering matrix of massless particles in non-abelian gauge theory. It was noticed by Witten [22] that the perturbative expansion of $\mathcal{N} = 4$ super Yang-Mills theory with U(N) gauge group is equivalent to the instanton expansion of a topological string theory whose target space is the Calabi-Yau supermanifold $\mathbb{C}P^{3|4}$. Using this string theory it was possible to reproduce the Maximal Helicity Violating Amplitudes at tree level for gluons in QCD. After this breakthrough and its generalization to include loops Britto et al. [6], a huge revolution for the computation of scattering amplitudes in non-abelian gauge theories has started, including the discovery of deeply connections between scattering of gluons with scattering of gravitons. Moreover, the progress in this direction have allowed to apply this techniques to the computation of realistic scattering amplitudes to be search at the LHC.

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http://dx.doi.org/10.1016/j.nuclphysbps.2015.10.098

Along the lines of this note we are going to comment on other stringy appearances into field theory. The particularity of the examples chose to be exposed comes form the similarities into the mechanism how extended objects appear, such as that they usually appear as a collective phenomena for a large charged set of states in the field theory as well as they are associated to non-perturbative effects.

Despite that we are going to see several different examples, we emphasize that we will be bearing in mind the AdS/CFT correspondence Maldacena [15] relating non-abelian conformal field theories with gravity theories (aka string theories). Since the original formulation of the correspondence Maldacena [15], an impressive amount of work have been done, including many tests of the duality as well as the exact formulation of it for non-abelian conformal field theories in dimensions $D = 2, 3, 4$. It also have been a big deal of progress in the application of the correspondence to more realistic scenarios, such as QCD or Condense Matter Systems.

2. Stringy behavior of the Hubbard Model

Let us start considering a simplified model which will share some characteristics with the gauge theories we are going to consider later on. Let us take a system which transform like a spin-s representation of SU(2) and interact with a external magnetic field B through a Zeeman-like potential $V_z = B \cdot S$, being S the spin-vector $S = (s_x, s_y, s_z)$, Fradkin [8]. The usual highest weight state representation is defined up to a normalization as the set of states diagonalizing at the same time both S_z, S^2,

$$S_z|s,s\rangle = s|s,s\rangle\,, \quad S^2|s,s\rangle = s(s+1)|s,s\rangle \tag{1}$$

The idea is to write the evolution operator of the system in terms of the following set of coherent states,

$$|n\rangle = \mathrm{e}^{i\theta(n_0\times n)\cdot S}|s,s\rangle\,, \tag{2}$$

where n_0 is a unitary vector in the z direction and θ is the angle between n_0 and the arbitrary unitary vector n. The above coherent state is an eigenvector of the spin operator $S|n\rangle = s|n\rangle$ and has an expansion in terms of the spins-s representation of SU(2) as,

$$|n\rangle = \sum_{m=-s}^{s} D^{(s)}(n)_{m\,s}|s,m\rangle\,, \tag{3}$$

the expansion coefficients satisfies the important property Perelomov [18],

$$D^{(s)}(n_1)D^{(s)}(n_2) = D^{(s)}(n_3)\mathrm{e}^{i\Phi(n_1,n_2,n_3)S_3}\,, \tag{4}$$

where $n_i, \quad i = 1, 2, 3$ are three arbitrary unit vectors and $\Phi(n_1, n_2, n_3)$ is the area enclosed by the spherical triangle with vertices at n_i. The normalization of the coherent states is,

$$\langle n_1|n_2\rangle = \mathrm{e}^{i\Phi(n_1,n_2,n_3)s}\left(\frac{1+n_1\cdot n_2}{2}\right)^s\,. \tag{5}$$

The completes relation in the space of coherent states is given in this case by,

$$\mathcal{I} = \left(\frac{2s+1}{4\pi}\right)\int d^3n\delta(n^2-1)|n\rangle\langle n|\,. \tag{6}$$

The above relations are sufficient to write the evolution operator

$$\begin{aligned} Z &= \mathrm{Tr}\mathrm{e}^{iV_zT} \sim \\ &\sim \lim_{N\to\infty}\left(\prod_{j=1}^{N}\int d\mu(n)\right)\left(\prod_{j=1}^{N}\langle n(t_j)|\mathrm{e}^{-\delta t V_z}|n(t_{j+1})\rangle\right) \\ &\sim \int \mathcal{D}n\,\mathrm{e}^{iS_M[n]}\,, \end{aligned} \tag{7}$$

where $d\mu(n) = \left(\frac{2s+1}{4\pi}\right)d^3n\,\delta(n^2-1)$ and the action is defined by

$$\begin{aligned} S_M[n] &= s\mathcal{A}[n] + m\int_0^T dx_0\,(\partial_0 n(x_0))^2 \\ &\quad -s\int_0^T dx_0 B\cdot n(x_0)\,, \end{aligned} \tag{8}$$

with $m = s\delta t/4 \to 0$. The $\mathcal{A}[n]$ corresponds to the area enclosed by the closed trajectory $n(x_0)$ on the unit-sphere surface, and can be rewritten as a Wess-Zumino action on the two-sphere. Hence the action (8) can be interpreted as a string on the surface of a sphere coupled to a $B-$field. Moreover, it scales with s and hence for large values of s, the action is dominated by the classical solutions of the string over the sphere. This stringy description should be replicated in higher dimensions as well as higher symmetrical systems.

3. Chern-Simons plus matter

Let us now consider a theory in dimension three which consist of double Chern-Simons matter theory in three dimension with level $(k, -k)$ and gauge group $U(N) \times U(N)$. The theory becomes weakly coupled when the level k is large, hence in the large N limit the coupling analogous to 't Hooft coupling is given by $\lambda \equiv N/k$, which is kept finite. The gauge fields are coupled to four chiral superfields in the bifundamental representation of the gauge group $U(N) \times U(N)$, and in the