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Radiative corrections to Lorentz-invariance violation with higher-order operators: Fine-tuning problem revisited

Sebastian Ossandon[a], Camilo Reyes[b], Carlos M. Reyes[c,*]

[a] *Instituto de Matemáticas, Pontificia Universidad Católica de Valparaíso, Casilla 4059, Valparaíso, Chile*
[b] *Departamento de Ciencias Fisicas, Facultad de Ciencias Exactas, Universidad Andres Bello, Republica 220, Santiago, Chile*
[c] *Departamento de Ciencias Básicas, Universidad del Bío Bío, Casilla 447, Chillán, Chile*

Abstract

We study the possible effects of large Lorentz violations that can appear in the effective models in which the Lorentz symmetry breakdown is performed with higher-order operators. For this we consider the Myers and Pospelov extension of QED with dimension-five operators in the photon sector and standard fermions. We focus on the fermion self-energy at one-loop order and find small and finite radiative corrections in the even CPT sector. In the odd CPT sector a lower dimensional operator is generated which contains unsuppressed effects of Lorentz violation leading to a possible fine-tuning. For the calculation of divergent diagrams we use dimensional regularization and consider an arbitrary background four-vector.

1. Introduction

New physics standing in the form of Lorentz symmetry violation has been a starting point for several effective models beyond the standard model [1]. A low energy remnant of this type is strongly motivated by the idea that spacetime changes drastically due to the appearance of some level or discreteness or spacetime foam at high energies. The effective approach has been shown to be extremely successful in order to contrast the possible Lorentz and CPT symmetry violations with experiments. A great part of these searches have been given within the framework of the standard model extension with several bounds on Lorentz symmetry violation provided [2, 3, 4]. In general most of the studies on Lorentz symmetry violation have been performed with operators of mass dimension $d \leq 4$, [5]. In part because the higher-order theories present some problems in their quantization [6]. However, in the last years these operators have received more attention and several bounds have been put forward [7, 8, 9, 10, 11]. Moreover, a generalization has been constructed to include non-minimal terms in the effective framework of the standard model extension [12].

Many years ago Lee-Wick [13] and Cutkosky [14] studied the unitarity of higher-order theories using the formalism of indefinite metrics in Hilbert space. They succeeded to prove that unitarity can be conserved in some higher-order models by restricting the space of asymptotic states. This has stimulated the construction of several higher-order models beyond the standard model [15]. One example is the Myers and Pospelov model based on dimension-five operators describing possible effects of quantum gravity [16, 17]. In the model the Lorentz symmetry violation is characterized by a preferred four-vector n [18, 19]. The preferred four-vector may be thought to come from a spontaneous symmetry breaking in an underlying fundamental theory. One of the original motivations to incorporate such terms was to produce cubic modifications in the dispersion relation, although an exact calculation yields a more complicated structure usually with the

*Corresponding author
Email address: creyes@ubiobio.cl (Carlos M. Reyes)

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gramian of the two vectors k and n involved. The Myers and Pospelov model has become an important arena to study higher-order effects of Lorentz-invariance violation [8, 20, 21, 22].

This work aims to contribute to the discussion on the fine-tuning problem due to Lorentz symmetry violation [23], in particular when higher-order operators are present. There are different approaches to the subject, for example using the ingredient of discreteness [24] or supersymmetry [25]. For renormalizable operators, including higher space derivatives, large Lorentz violations can or cannot appear depending on the model and regularization scheme [26]. However, higher-order operators are good candidates to produce strong Lorentz violations via induced lower dimensional operators [27]. Some attempts to deal with the fine tuning problem considers modifications in the tensor contraction with a given Feynman diagram [16] or just restrict attention to higher-order corrections [28]. However in both cases the problem comes back at higher-order loops [29]. Here we analyze higher-order Lorentz violation by explicitly computing the radiative corrections in the Myers and Pospelov extension of QED. We use dimensional regularization which eventually preserves unitarity, thus extending some early treatments [18, 20].

2. Lorentz fine-tuning

Consider the Yukawa model [23, 30]

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\mu^2\phi^2 + \bar{\psi}(i\partial\!\!\!/ - m)\psi + g_Y\bar{\psi}\phi\psi\ , \quad (1)$$

where the Lorentz violation was implemented by modifying the propagators. In particular, the fermion propagator changes as

$$\frac{i}{k\!\!\!/ - m + i\epsilon} \longrightarrow \frac{i f\left(|\vec{k}|/\Lambda\right)}{k\!\!\!/ - m + i\epsilon}\ , \quad (2)$$

where Λ is an explicit cutoff and $f\left(|\vec{k}|/\Lambda\right)$ obeys $f(0) = 1$ and $f(\infty) = 0$. In this way the scalar self-energy $\Pi(p)$ is given by

$$\Pi(p) = -ig_Y^2 \int \frac{d^4k}{(2\pi)^4} f\left(|\vec{k}|/\Lambda\right) f\left(|\vec{k}+\vec{p}|/\Lambda\right) \times \mathrm{Tr}\left[\frac{1}{(k\!\!\!/ - m + i\epsilon)}\frac{1}{(k\!\!\!/ + p\!\!\!/ - m + i\epsilon)}\right]. \quad (3)$$

The above integral has been ultraviolet regularized and is therefore convergent. We expect to recover the usual divergencies in the limit $\Lambda \to 0$ of the first terms of the expansion around $p = 0$

$$\Pi(p) = \Pi(0) + \frac{p_0 p_0}{2!}\left(\frac{\partial^2\Pi(p)}{\partial p_0 \partial p_0}\right)_{p=0} + \frac{p_i p_i}{2!}\left(\frac{\partial^2\Pi(p)}{\partial p_i \partial p_i}\right)_{p=0} + \text{conv}\ . \quad (4)$$

Above we have considered that the mixed terms

$$\frac{\partial\Pi(0)}{\partial p_\mu}, \qquad \frac{\partial^2\Pi(0)}{\partial p_0 \partial p_i}, \qquad \frac{\partial^2\Pi(0)}{\partial p_i \partial p_j}\ , \quad (5)$$

vanish. We have from rotational invariance

$$\Pi(p) = \Pi(0) + \frac{1}{2!}(p_0^2 - \vec{p}^2)\left(\frac{\partial^2\Pi(p)}{\partial p_1 \partial p^1}\right)_{p=0} + p_0^2\,\eta_{LV} + \Pi_\Lambda\ , \quad (6)$$

where the Lorentz violation is parametrized by the quantity

$$\eta_{LV} = \frac{1}{2!}\left(\frac{\partial^2\Pi(p)}{\partial p_0 \partial p_0} + \frac{\partial^2\Pi(p)}{\partial p_1 \partial p_1}\right)_{p=0}\ . \quad (7)$$

After some calculation, both contributions give

$$\left(\frac{\partial^2\Pi(p)}{\partial p_0 \partial p_0} + \frac{\partial^2\Pi(p)}{\partial p_1 \partial p_1}\right)_{p=0} = -ig_Y^2\int\frac{d^4k}{(2\pi)^4} \times f(|\vec{k}|/\Lambda)\left(-2\left(\frac{\partial f\left(|\vec{k}+\vec{p}|/\Lambda\right)}{\partial p_1}\right)_{p=0} \times \mathrm{Tr}\left[\frac{1}{(k\!\!\!/ - m)}\gamma^1\frac{1}{(k\!\!\!/ - m)^2}\right] + \left(\frac{\partial^2 f\left(|\vec{k}+\vec{p}|/\Lambda\right)}{\partial p_1^2}\right)_{p=0}\mathrm{Tr}\left[\frac{1}{(k\!\!\!/ - m)^2}\right]\right). \quad (8)$$

The dominant term can be obtained by setting $m = 0$

$$\eta_{LV} = -2ig_Y^2\int\frac{d^4k}{(2\pi)^4}\frac{f(|\vec{k}|/\Lambda)}{k^2} \times \left(\frac{\partial^2 f(|\vec{k}+\vec{p}|/\Lambda)}{\partial p_1^2}\right)_{p=0}. \quad (9)$$

Integrating in k_0, and changing $\left(\frac{\partial^2 f\left(\frac{|\vec{k}+\vec{p}|}{\Lambda}\right)}{\partial p_1^2}\right)_{p=0} = \frac{\partial^2 f\left(\frac{|\vec{k}|}{\Lambda}\right)}{\partial k_1^2}$, we have

$$\eta_{LV} = -\frac{2g_Y^2}{16\pi^3}\int\frac{d^3k}{(2\pi)^4}\frac{1}{|\vec{k}|}f\left(|\vec{k}|/\Lambda\right)\frac{\partial^2 f(|\vec{k}|/\Lambda)}{\partial k_1^2}\ , \quad (10)$$

and using $\frac{\partial^2 f\left(|\vec{k}|/\Lambda\right)}{\partial k_1^2} = \frac{f''(x)}{3\Lambda^2} + \frac{2f'(x)}{3x\Lambda^2}$ inside the integral where the derivatives are with respect to $x = |\vec{k}|/\Lambda$, we arrive at

$$\eta_{LV} = \frac{g_Y^2}{12\pi^2}\left(1 + 2\int_0^\infty dx\, x\,(f'(x))^2\right)\ . \quad (11)$$