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Towards a Numerical Implementation of the Loop-Tree Duality Method

Sebastian Buchta[a,*], Grigorios Chachamis[a], Petros Draggiotis[b], Ioannis Malamos[a], Germán Rodrigo[a]

[a] Instituto de Física Corpuscular, Universitat de València - Consejo Superior de Investigaciones Científicas, Parc Científic, E-46980 Paterna, Valencia, Spain

[b] Institute of Nuclear and Particle Physics, NCSR "Demokritos", Agia Paraskevi, 15310, Greece

Abstract

We review the recent progress on the numerical implementation of the Loop-Tree Duality Method (LTDM) for the calculation of scattering amplitudes. A central point is the analysis of the singularities of the integrand. In the framework of the LTDM some of these singularities cancel out. The ones left over are dealt with by contour deformation. We present details on how to achieve this as well as first results.

1. Introduction

When calculating NLO (NNLO) cross-sections one needs to consider the tree- and loop-contributions separately. Especially loops with many external legs prove to be challenging. Considerable progress has already been made in order to attack this problem: OPP-Method, Unitarity Methods, Mellin-Barnes Representation, Sector Decomposition [1]. The advantage of these methods is that they made possible what was impossible before, but still a lot of effort has to be put in to cancel infrared singularities among real and virtual corrections. Additional difficulties arise from threshold singularities that lead to numerical instabilities. The Loop-Tree Duality method aims towards a combined treatment of tree- and loop- contributions. Therefore the Loop-Tree Duality method casts the virtual corrections in a form that closely resembles the real ones.

2. Loop-Tree Duality at one loop

The most general, dimensionally regularized one-loop scalar integral can be written as [2]:

$$L^{(1)}(p_1, p_2, \dots, p_N) = \int_{\ell_1} \prod_{i=1}^{N} G_F(q_i) \qquad (1)$$

with the Feynman propagator $G_F(q_i) = [q_i^2 - m_i^2 + i0]^{-1}$, internal momenta $q_i = \ell_1 + p_1 + \cdots + p_i = \ell_1 + k_i$ and shorthand integral notation $\int_{\ell_1} = -i \int d^d\ell_1/(2\pi)^d$. As a first step, one performs the integration over the complex energy components of the loop four-momentum by applying the residue theorem. The integration contour is chosen such that it encloses the poles with positive energy and negative imaginary part, see figure below:

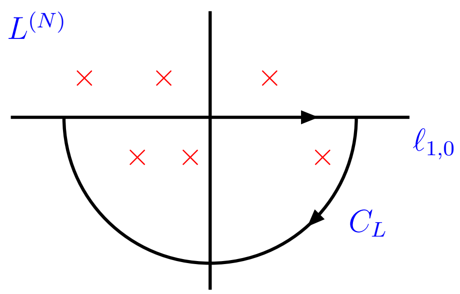

Figure 1: Location of poles and integration contour C_L in the complex $\ell_{1,0}$-plane.

The residue theorem is employed by taking the residues of the poles inside of the contour and summing over them. Given an appropriate gauge choice the integrand in eq. (1) contains only simple poles. Thus the residue of an individual pole is done by taking the residue of a single propagator and evaluating the other propagators at the position of the residue

$$\mathrm{Res}_{\mathrm{Im}\{q_{i,0}\}<0} \frac{1}{q_i^2 - m_i^2 + i0} = \int d\ell_1 \delta_+(q_i^2 - m_i^2) \qquad (2)$$

*Speaker

Email addresses: sebastian.buchta@ific.uv.es (Sebastian Buchta), grigorios.chachamis@ific.uv.es (Grigorios Chachamis), petros.draggiotis@gmail.com (Petros Draggiotis), ioannis.malamos@ific.uv.es (Ioannis Malamos), german.rodrigo@csic.uv.es (Germán Rodrigo)

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$$\prod_{j\neq i} G_F(q_j)\Bigg|_{\text{i-th pole}} = \prod_{j\neq i} \frac{1}{q_j^2 - m_j^2 - i0\eta(q_j - q_i)} \equiv \prod_{j\neq i} G_D(q_i;q_j). \quad (3)$$

The subscript "+" on the right hand side of eq. (2) indicates that the positive-energy solution is to be taken. Furthermore η is a future-like vector, i.e. $\eta^2 \geq 0$, $\eta_0 > 0$. It is dependent on the choice of coordinate system, however it cancels out once one adds all dual contributions. Hence physical objects like scattering cross sections will stay frame-indepedent. Evaluating the "non-cut" propagators at the position of the pole leads to a modification of the usual Feynman prescription. In eq. (3) it is shown that instead one ends up with the so called "dual prescription" which serves to keep track of the correct sign of the i0-prescription of the corresponding propagator. Collecting all the pieces and putting them together, one arrives at

$$L^{(1)}(p_1, p_2, \ldots, p_N) = -\sum \int_{\ell_1} \tilde{\delta}(q_i) \prod_{\substack{j=1 \\ j\neq i}}^{N} G_D(q_i;q_j) \quad (4)$$

with $\tilde{\delta}(q_i) = 2\pi i \delta_+(q_i^2 - m_i^2)$. Thus, by virtue of employing the residue theorem, it is possible to rewrite a one-loop amplitude as a sum of single-cut phase-space integrals over the loop-three-momentum. The i-th dual contribution has the i-th propagator set on-shell while the left over Feynman propagators get promoted to Dual propagators.

$$L^{(1)}(p_1, p_2, \ldots, p_N) =$$

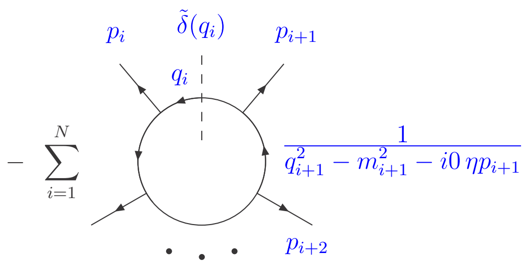

Figure 2: Graphical representation of the solution of the LTDM at one-loop.

The LTDM features a couple of interesting properties:

- Number of single cut Dual Contributions equals the number of legs, this way a loop diagram is fully opened to tree diagrams.
- The singularities of the loop diagram appear as singularities of the Dual Integrals.
- Tensor loop integrals and physical scattering amplitudes are treated in the same way since the Loop-Tree Duality works only on propagators.
- Virtual corrections are recast in a form, that closely parallels the contribution of real corrections.

This is the formalism for the one-loop case. Solutions for more complicated situations like multiple loops [3] or higher order poles [4] are described in the respective references.

3. Singular behavior of the loop integrand

As a preparatory step it will prove useful to introduce an alternative way of denoting the dual propagator. This will give a more natural access to its singularities.

$$\tilde{\delta}(q_i) G_D(q_i;q_j) = 2\pi i \frac{\delta(q_{i,0} - q_{i,0}^{(+)})}{2q_{i,0}^{(+)}} \frac{1}{(q_{i,0}^{(+)} + k_{ji,0})^2 - (q_{j,0}^{(+)})^2} \quad (5)$$

with $k_{ji} = q_j - q_i$ and $q_{i,0}^{(+)} = \sqrt{\mathbf{q_i}^2 + m_i^2 - i0}$.
In fig. (3), the on-shell hyperboloids of three propagators in loop-mometum-space are sketched.

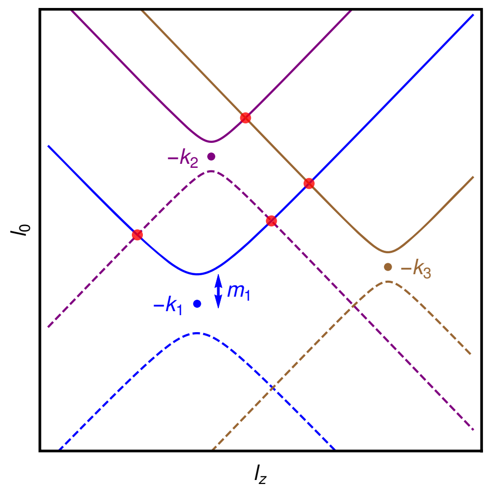

Figure 3: On-shell hyperboloids for three arbitrary propagators in Cartesian coordinates.

The loop integrand becomes singular at hyperboloids with $q_{i,0}^{(+)} = \sqrt{\mathbf{q_i}^2 + m_i^2 - i0}$ (solid lines) and $q_{i,0}^{(-)} = -\sqrt{\mathbf{q_i}^2 + m_i^2 - i0}$ (dashed lines) and origin in $-k_{i,\mu}$. Applying the LTDM is equivalent to integrating along the