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Testing different formulations of leading-order anisotropic hydrodynamics

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Abstract

A recently obtained set of the equations for leading-order (3+1)D anisotropic hydrodynamics is tested against exact solutions of the Boltzmann equation with the collisional kernel treated in the relaxation time approximation. In order to perform detailed comparisons, the new anisotropic hydrodynamics equations are reduced to the boost-invariant and transversally homogeneous case. The agreement with the exact solutions found using the new anisotropic hydrodynamics equations is similar to that found using previous, less general formulations of anisotropic hydrodynamics. In addition, we find that, when compared to a state-of-the-art second-order viscous hydrodynamics framework, leading-order anisotropic hydrodynamics better reproduces the exact solution for the pressure anisotropy and gives comparable results for the bulk pressure evolution. Finally, we compare the transport coefficients obtained using linearized anisotropic hydrodynamics with results obtained using second-order viscous hydrodynamics.

Keywords: Relativistic heavy-ion collisions; Quark–gluon plasma; Anisotropic dynamics; Viscous hydrodynamics; Boltzmann equation; Relaxation time approximation

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1. Introduction

The successful explanation of the space-time evolution of matter produced in relativistic heavy-ion collisions at RHIC (Relativistic Heavy-Ion Collider) and the LHC (Large Hadron Collider) using relativistic viscous hydrodynamics initiated new developments of the hydrodynamics [1–30]. An example of such an activity is the burgeoning study of *anisotropic hydrodynamics*. In its original formulation, anisotropic hydrodynamics was restricted to boost-invariant and transversally homogeneous systems [31,32], denoted below as the (0+1)D case. Shortly afterwards, this approach was generalized to the one-dimensional, non-boost-invariant situation [33,34] and to the case of radial expansion [35]. The (2+1)D and (3+1)D cases (corresponding to a two-dimensional transverse expansion with longitudinal boost-invariance and to an arbitrary three-dimensional expansion without any symmetry constraints, respectively) were studied in a subsequent series of papers [36–39]. In parallel, the concept of anisotropic hydrodynamics was extended to describe mixtures of anisotropic quark and gluon fluids [40–42].

In Refs. [31–34,36–42] only the leading order of the hydrodynamic expansion has been included and described by the spheroidal Romatschke–Strickland form [43]. This form allows for the difference between the longitudinal and transverse pressures only (for transversally homogeneous systems) and hence cannot describe more complex situations where, for example, the three components of pressure differ from each other. A successful method to solve this problem by including further terms in the hydrodynamic expansion was first presented in Ref. [44] for massless (conformal) systems and was recently extended to massive (non-conformal) systems in Ref. [45].

An alternative starting point for performing the anisotropic hydrodynamics expansion is to include the momentum anisotropy in a more complete way already at leading order by generalizing the Romatschke–Strickland form to allow the energy-momentum tensor to possess different pressures along all three spatial directions in the local rest frame (ellipsoidal form). Such a non-perturbative treatment for the (1+1)D case (boost-invariant expansion with cylindrical spatial symmetry) was proposed for conformal systems in Ref. [46] and soon generalized to non-conformal systems in Ref. [47]. Recently, a more general set of equations for leading-order anisotropic hydrodynamics has been presented by Tinti in Ref. [48]. Differently from previous leading-order approaches, the new formulation does not use any simplifying symmetry assumptions such as the longitudinal boost invariance or cylindrical symmetry. Using this new approach it was shown in [48] that anisotropic hydrodynamics reproduces the structure of second-order viscous hydrodynamics equations [24].

Nevertheless, one finds that the latest anisotropic hydrodynamics formulation obtained in Ref. [48] allows for several different ways to obtain the equations necessary to describe the evolution of the bulk pressure of the system. In order to determine which option is the best for possible phenomenological applications, in this paper we test the corresponding solutions with the help of exact solutions of the Boltzmann equation treated in the relaxation time approximation. To perform the numerical comparisons, the new equations are reduced to the boost-invariant and transversally homogeneous case, where the solutions of the Boltzmann equation are available. We note that comparisons of the predictions of various hydrodynamic frameworks with the solutions of the underlying kinetic theory has now become a quite useful method for testing the accuracy of different hydrodynamic formulations [49–55].

In this paper, we show that the agreement of the new formulation of anisotropic hydrodynamics with the exact solutions is similar to that obtained using earlier, less general, formulations of anisotropic hydrodynamics and comparable with the best prescriptions of the second-order