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Mapping between the classical and pseudoclassical models of a relativistic spinning particle in external bosonic and fermionic fields. I

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Abstract

The problem on mapping between two Lagrangian descriptions (using a commuting c-number spinor ψ_α or anticommuting pseudovector ξ_μ and pseudoscalar ξ_5 variables) of the spin degrees of freedom of a color spinning massive particle interacting with background non-Abelian gauge field, is considered. A general analysis of the mapping between a pair of *Majorana* spinors $(\psi_\alpha, \theta_\alpha)$ (θ_α is some auxiliary anticommuting spinor) and a *real anticommuting* tensor aggregate $(S, V_\mu, {}^*T_{\mu\nu}, A_\mu, P)$, is presented. A complete system of bilinear relations between the tensor quantities, is obtained. The analysis we have given is used for the above problem of the equivalence of two different ways of describing the spin degrees of freedom of the relativistic particle. The mapping of the kinetic term $(i\hbar/2)(\bar{\theta}\theta)(\dot{\bar{\psi}}\psi - \bar{\psi}\dot{\psi})$, the term $(1/e)(\bar{\theta}\theta)\dot{x}_\mu(\bar{\psi}\gamma^\mu\psi)$ that provides a couple of the spinning variable ψ and the particle velocity $\dot{x}_\mu$, and the interaction term $\hbar(\bar{\theta}\theta)Q^a F^a_{\mu\nu}(\bar{\psi}\sigma^{\mu\nu}\psi)$ with an external non-Abelian gauge field, are considered in detail. In the former case a corresponding system of bilinear identities including both the tensor variables and their derivatives $(\dot{S}, \dot{V}_\mu, {}^*\dot{T}_{\mu\nu}, \dot{A}_\mu, \dot{P})$, is defined. A detailed analysis of the local bosonic symmetry of the Lagrangian with the commuting spinor ψ_α, is carried out. A connection of this symmetry with the local SUSY transformation of the Lagrangian containing anticommuting pseudovector and pseudoscalar variables, is considered. The approach of obtaining a supersymmetric Lagrangian in terms of the even ψ_α and odd θ_α spinors, is offered.

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1. Introduction

In our paper [1] the model Lagrangian describing the interaction of a relativistic spinning color-charged classical particle with background non-Abelian gauge and fermion fields was suggested. The spin degrees of freedom have been presented in [1] by a c-number Dirac spinor ψ_α, $\alpha = 1, \dots, 4$. By virtue of the fact that the background fermion field $\Psi^i_\alpha(x)$ (which within the classical description is considered as a Grassmann-odd function) has, by definition, the spinor index α, the description of the spin degrees of freedom of the particle in terms of the spinor ψ_α is very natural and simplest in technical respect. There is some vagueness with respect to Grassmann parity of this spinor. In our papers [2,3] in application to an analysis of dynamics of a spinning color particle moving in a hot quark–gluon plasma, the spinor ψ_α was thought as the Grassmann-even parity one (although it is not improbable that simultaneous using of spinors of different Grassmann parity may be required for a complete classical description of the spin dynamics in external fields of different statistics). Furthermore, for simplicity throughout our previous works [2–6], we have neglected a change of the spin state of the particle, i.e. we believed ψ_α to be a spinor independent of the evolution parameter τ. As a result we have completely neglected an influence of the spin of particle on the general dynamics of the interaction of the particle with background fields. However, for a more detailed study of the motion of a particle in external fields of different statistics and comparing the suggested model with the other approaches known in the literature, it is necessary to account for a change in time of the spin variable ψ_α. At present there exist a few approaches to the description of the spin degrees of freedom of a particle within the (semi)classical approximation. Below only two approaches closely related to the subject of our subsequent investigation are outlined.

Notice that the description of the spin degrees of freedom by means of a classical commuting spinor is not new. Such a way of the description arises naturally in determining the connection of relativistic quantum mechanics of an electron with relativistic classical mechanics [7]. In particular, it was shown [8–10] that within the WKB-method extended to the relativistic case, the relativistic wave Dirac equation results in a system of equations incorporating not only the classical canonical equations of motion, but also yet another equation for the spin degrees of freedom. This equation is connected directly with the Schrödinger equation

$$i\hbar \frac{d\psi(\tau)}{d\tau} = -\frac{q\hbar}{4m}\,\sigma^{\mu\nu} F_{\mu\nu}(x)\psi(\tau) \tag{1.1}$$

for the commuting spinor function ψ_α (we put throughout $c = 1$ for the speed of light). Here, $\sigma^{\mu\nu} = [\gamma^\mu, \gamma^\nu]/2i$ and q is an electric charge. This equation describes the motion of the spin of the electron in a given electromagnetic field $F_{\mu\nu}(x)$. The field in (1.1) is defined along the path of particle $x_\mu = x_\mu(\tau, x_0, \tau_0)$ in four-dimensional Minkowski space ('mostly minus' metric), as a function of the proper time τ.

Further, Bohm et al. [11] have introduced two-component spinor in *classical* (non-relativistic) hydrodynamics in view of obtaining a causal model for the Pauli equation. Their method consists in associating the spinor with the rotation of an element of the fluid. Unfortunately, this method breaks down in point mechanics and it is difficult to extend it to the relativistic case. Another line of thought is due to Proca [12] who attaches a bispinor to a point particle. He then proceeds to find