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Time as a dynamical variable in quantum decay

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Abstract

We present a theoretical analysis of quantum decay in which the survival probability is replaced by a decay rate that is equal to the absolute value squared of the wave function in the time representation. The wave function in the time representation is simply the Fourier transform of the wave function in the energy representation, and it is also the probability amplitude generated by the Positive Operator Valued Measure of a time operator. The present analysis endows time with a dynamical character in quantum decay, and it is applicable only when the unstable system is monitored continuously while it decays. When the analysis is applied to the Gamow state, one recovers the exponential decay law. The analysis allows us to interpret the oscillations in the decay rate of the GSI anomaly, of neutral mesons, and of fluorescence quantum beats as the result of the interference of two resonances in the time representation. In addition, the analysis allows us to show that the time of flight of a resonance coincides with its lifetime.

Keywords: Gamow states; Resonances; Time operators; Time of flight; Continuous measurements; Zeno effect

1. Introduction

In quantum mechanics, time plays the role of an external parameter, and therefore it is apparently not a dynamical variable, as made clear by Pauli's theorem [1]. However, there are many experimental situations such as the time of flight or the decay of an unstable particle in which time seems to play a dynamical role. For example, the lifetime of a particle seems to be an intrinsic dynamical property of the particle, not just a mere parameter.

Many authors have constructed time operators that endow time with a dynamical character, see for example Refs. [2–15] and references therein. Such time operators are usually [2–11]

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associated with Positive Operator Valued Measures (POVMs) and therefore circumvent Pauli's theorem. POVMs not only provide a natural setting for time operators, but also for phase operators and for the momentum operator of a one-dimensional particle on the half line. Rather than being uncommon, POVMs are standard tools in the quantum theory of open systems [16] and in quantum information and computation [17,18].

Although the mathematical aspects of the POVMs associated with time operators are well established, their phenomenological signatures have remained elusive [19]. The purpose of this paper is to propose a theoretical analysis of quantum decay in which the decay rate is given by the probability distribution associated with the POVM of a time operator. In such analysis, time appears explicitly as a dynamical variable (or, more precisely, as a random variable). We will show that the probability distribution associated with the POVM of the time operator is different from the survival probability. We will also show that the time representation of the Gamow states describes the exponential region of quantum decay while explicitly displaying the dynamical character of time.

As we will stress along the paper, describing the decay on an unstable system in the time representation is necessary only in experiments that monitor the system's decay continuously. One such experiment is the so-called GSI anomaly [20], where Litvinov et al. observed that K-shell electron capture decay rates of Hydrogen-like $^{140}\mathrm{Pr}^{58+}$ and $^{142}\mathrm{Pm}^{60+}$ ions show an oscillatory modulation superimposed on the exponential decay. Because Litvinov et al. monitored individual ions continuously, we will interpret the GSI anomaly as the result of the interference of two resonances in the time representation. We will also see that such interpretation could be applied to the decay of K and B mesons and to fluorescence quantum beats if the decay of these systems were monitored continuously.

In Section 2, we recall the basic phenomenological features of exponential decay. In Section 3, we recall the standard theoretical analysis of quantum decay. In Section 4, we construct the time representation and use dimensional analysis to identify the decay rate with the absolute value squared of the wave function in the time representation. In Sections 5 and 6, we obtain the time representation of a Gamow state and show that such time representation accounts for the phenomenology of exponential decay. In Section 7, we compare the survival probability $p_{\mathrm{s}}(\tau)$ with the non-decay probability $\mathcal{P}(t)$ associated with the time representation, and we point out that $\mathcal{P}(t)$ does not exhibit the Zeno effect. In Sections 8 and 9, we show that the interference of two resonances in the time representation can account for the GSI anomaly, for fluorescence quantum beats, and for the decay of neutral mesons. In Section 10, we compare the pulsed and the continuous measurements of the survival probability $p_{\mathrm{s}}(\tau)$ with the measurement of the non-decay probability $\mathcal{P}(t)$, and we argue that the measurement of $\mathcal{P}(t)$ is inherently continuous. In Section 11, we use the time representation to derive an expression for the time of flight of a particle, and we show that the time of flight of a resonance is equal to its lifetime, as it is usually assumed. Section 12 contains our conclusions.

2. Phenomenology of radioactive decay

The standard phenomenological treatment of the decay of a radioactive sample is as follows. When a sample of radioactive nuclei contains $N(t)$ radioactive nuclei at time t, the rate at which nuclei decay is proportional to $N(t)$

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = -\lambda N(t), \tag{2.1}$$