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Spectral functions from the functional renormalization group

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Abstract

In this article we wish to present a new method to obtain spectral functions at finite temperature and density from the Functional Renormalization Group (FRG). The FRG offers a powerful non-perturbative tool to deal with phase transitions in strong-interaction matter under extreme conditions and their fluctuation properties. Based on a thermodynamically consistent truncation we derive flow equations for pertinent two-point functions in Minkowski space–time. We demonstrate the feasibility of the method by calculating mesonic spectral functions in hot and dense hadronic matter using the quark–meson model as a simple example.

Keywords: Spectral function; Analytic continuation; QCD phase diagram

1. Introduction

The in-medium modifications of hadron properties have been dear to Gerry's heart and his work on this subject has made him and his collaborators major drivers of this field for several

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decades [1,2]. The discussions are far from over and we wish to pay tribute to Gerry's physical insights by adding a new and promising alternative for computing real-time spectral functions in hot and dense QCD matter. The aim is to set up a framework that goes beyond mean-field theory (which Gerry employed very successfully during his career) by incorporating quantum fluctuations. This is particularly important for the understanding of the restoration of broken chiral symmetry in the hadronic medium.

Spectral functions encode information on the particle spectrum as well as collective excitations of a given system. A thermodynamically consistent calculation beyond the Hartree–Fock level of such real-time observables represents an inherently difficult problem. Although mean-field calculations might capture the gross features of the equilibrium properties, quantitative predictions and correct descriptions of critical phenomena require the proper inclusion of fluctuations. Several self-consistent methods are available among which the Functional Renormalization Group (FRG) is particularly suitable and widely used in quantum field theory and condensed matter physics [3–9].

A technical difficulty which is common to all Euclidean approaches to Quantum Field Theory is the need to analytically continue from imaginary to real time especially for dynamic processes with time-like momentum transfers. At finite temperature these continuations are often based on numerical (i.e. noisy) data at discrete Matsubara frequencies and several approximate methods for the reconstruction of real-time spectral functions have been used [10–13] with varied success. Therefore any approach that can deal with the analytic continuation explicitly is highly desirable. Such alternative approaches have been proposed in [14,15] and [16]. They involve an analytic continuation on the level of the FRG flow equations for two-point correlation functions and have been applied in [15] to a model system with O(4) internal symmetry in vacuum. More recently, this method has been applied to obtain spectral functions from the quark–meson model at finite temperature and density [17]. These studies form the basis for the present discussion of hadronic matter at high temperatures and large baryo-chemical potentials.

2. The functional renormalization group

To set the stage we start out with the basic ideas of the FRG, by discussing the effective potential of a system in thermal equilibrium, the ensuing correlation functions and their flow equations.

2.1. Effective action

The principal object of statistical physics is the (Euclidean) partition function Z from which the equilibrium properties of a thermal system can be derived. Z can be represented as a Feynman path integral (for simplicity in a real field variable ϕ). In the presence of an external source j it reads:

$$Z[j] = e^{W[j]} = \int [\mathcal{D}\phi] e^{-S[\phi]+\int d^4x\, \phi(x) j(x)}, \tag{1}$$

and $W[j]$ generates all n-point Green functions via functional derivatives w.r.t. the source function j. In particular

$$\left.\frac{\delta W[j]}{\delta j(x)}\right|_{j=0} = \langle \phi(x) \rangle \equiv \varphi(x) \tag{2}$$