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Fermi liquid theory: A brief survey in memory of Gerald E. Brown

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Abstract

I present a brief review of Fermi liquid theory, and discuss recent work on Fermi liquid theory in dilute neutron matter and cold atomic gases. I argue that recent interest in transport properties of quantum fluids provides fresh support for Landau's approach to Fermi liquid theory, which is based on kinetic theory rather than effective field theory and the renormalization group. I also discuss work on non-Fermi liquids, in particular dense quark matter.

Keywords: Many-body physics; Transport theory

1. Introduction

One of Gerry's main scientific pursuits was to understand the nuclear few- and many-body problem in terms of microscopic theories based on the measured two- and three-nucleon forces. One of the challenges of this program is to understand how the observed single-particle aspects of finite nuclei, in particular shell structure and the presence of excited levels which carry the quantum numbers of single particle states, can be reconciled with the strong nucleon–nucleon force, and how single particle states can coexist with collective modes. A natural framework for addressing these questions is the Landau theory of Fermi liquids. Landau Fermi liquid theory describes a, possibly strongly correlated, Fermi system which is adiabatically connected to a free Fermi gas. In particular, the system has a Fermi surface, and the excitations are quasi-particles with the quantum numbers of free fermions, but with modified dispersion relations and effective interactions. These quasi-particles coexist with collective modes, for example zero sound.

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Gerry reviewed Fermi liquid theory in a number of his books and other writings. The conference that celebrated his 60th birthday was titled "Windsurfing the Fermi Sea" [1]. In the introduction of *Unified Theory of Nuclear Models and Forces* (3rd edition, 1970) Gerry writes:

> *Many improvements could have been made, especially in Chapter XIII on effective forces in nuclei, but time is short, and I shall make them in later editions, when I am too old to ski. Of course, nobody will be interested in the subject by then.*

This prediction turned out to be incorrect. In his final decade at Stony Brook Gerry trained and mentored a remarkable group of students who have helped to reinvigorate the study of effective forces in nuclei [2,3].

2. Landau Fermi liquid theory

Consider a cold Fermi system in which the low energy excitations are spin $1/2$ quasi-particles. Landau proposed to define a distribution function $f_p = f_p^0 + \delta f_p$ for the quasi-particles. Here, f_p^0 is the ground state distribution function, and $\delta f_p \ll f_p^0$ is a small correction. The energy density can be written as [4]

$$\mathcal{E} = \mathcal{E}_0 + \int d\Gamma_p \frac{\delta\mathcal{E}}{\delta f_p}\delta f_p + \frac{1}{2}\int\int d\Gamma_p d\Gamma_{p'} \frac{\delta^2\mathcal{E}}{\delta f_p \delta f_{p'}}\delta f_p \delta f_{p'} + \ldots, \tag{1}$$

with $d\Gamma_p = d^3p/(2\pi)^3$. Functional derivatives of $\mathcal{E}$ with respect to f_p define the quasi-particle energy E_p and the effective interaction $t_{pp'}$

$$E_p = \frac{\delta\mathcal{E}}{\delta f_p}, \qquad t_{pp'} = \frac{\delta^2\mathcal{E}}{\delta f_p \delta f_{p'}}. \tag{2}$$

Near the Fermi surface we can write $E_p = v_F(|\vec{p}| - p_F)$, where v_F is the Fermi velocity, p_F is the Fermi momentum, and $m^* = p_F/v_F$ is the effective mass. We can decompose $t_{pp'} = F_{pp'} + G_{pp'}\vec{\sigma}_1 \cdot \vec{\sigma}_2$. On the Fermi surface the effective interaction is only a function of the scattering angle and we can expand the angular dependence as

$$F_{pp'} = \sum_l F_l P_l(\cos\theta_{\vec{p}\cdot\vec{p}'}), \tag{3}$$

where $P_l(x)$ is a Legendre polynomial, and $G_{pp'}$ can be expanded in an analogous fashion. The coefficients F_l and G_l are termed Landau parameters.

The distribution function satisfies a Boltzmann equation

$$(\partial_t + \vec{v}_p \cdot \vec{\nabla}_x + \vec{F}_p \cdot \vec{\nabla}_p) f_p(x,t) = C[f_p] \tag{4}$$

where $\vec{v}_p = \vec{\nabla}_p E_p$ is the quasi-particle velocity, $\vec{F}_p = -\vec{\nabla}_x E_p$ is an effective force, and $C[f_p]$ is the collision term. Conserved currents can be defined in terms of f_p and the single particle properties E_p and v_p. For example, we can write the mass density ρ and mass current $\vec{\jmath}$ as

$$\rho = \int d\Gamma_p m f_p, \qquad \vec{\jmath} = \int d\Gamma_p m\vec{v}_p f_p, \tag{5}$$

where $d\Gamma_p = d^3p/(2\pi)^3$. The Boltzmann equation implies that the current is conserved, $\partial_0\rho + \vec{\nabla}\cdot\vec{\jmath} = 0$. The conditions given in Eq. (2) play an important is proving conservation laws for energy and momentum, and in establishing sum rules.