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Many-body physics: Collective fermionic excitations in quark–gluon plasmas and cold atom systems

Jean-Paul Blaizot

Institut de Physique Théorique, CNRS/URA2306, CEA-Saclay, 91191 Gif-sur-Yvette, France

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Abstract

In this talk I discuss collective excitations that carry fermion quantum numbers. Such excitations occur in the quark–gluon plasma and can also be produced in cold atom systems under special conditions.

Keywords: Quark–gluon plasma; QCD; Collective excitations

1. Introduction

My first memories of Gerry bring me back to my very first year in physics. Indeed I first met Gerry as I was just starting my PhD. He used to visit Saclay regularly then, and everyone in the nuclear theory group was eager to show him his/her last results, and get his opinion. Certainly, like everybody else, I was also under the charm of his exceptional personality, and was indeed surprised that he would pay any attention to me and show curiosity for what I was doing. At the same time, I must confess that it took me some time to truly appreciate Gerry's way of doing physics. My academic training, and the particular environment of the IPhT, where mathematical rigor and elegance were somewhat more praised than physical intuition, did not help me there. However, with time, I had many occasions to be impressed by Gerry's intuition, his art of formulating good questions and distributing interesting problems to his (many) young collaborators, and his ability of getting deep physical insight into a complicated problem through a simple, but to the point, calculation.

When I first met Gerry, his book, *Many-Body Physics*, had just appeared [1]. This book has accompanied me during my first years of research. I learned from it, asides from the many

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discussions I have had with Gerry, about the basics of "traditional" many body physics: Fermi liquid theory, collective excitations in a variety of systems and the Random Phase Approximation, Brueckner theory, etc. It is with all this in mind that I have been preparing this talk. I have chosen to focus on collective excitations of a somewhat peculiar nature, since they carry fermionic degrees of freedom (most collective modes we are familiar with have bosonic character). I shall start with the quark gluon plasma, where fermionic excitations have been identified (theoretically!) long ago. I shall discuss the very long wavelength modes that can be interpreted as Goldstone excitations associated to a broken (approximate) supersymmetry. Then I shall move to cold atom systems, where efforts are being made to produce analogous phenomena.

2. Quark–gluon plasma

Let us then consider a quark–gluon plasma. I shall assume the temperature sufficiently high for the coupling constant g to be small. (I therefore leave aside interesting issues related to so-called strongly coupled quark–gluon plasma, the AdS/CFT correspondence, etc.) The basic degrees of freedom in such a system are quarks and gluons quasiparticles. From the point of view of many body physics, such systems are interesting because the long range interactions favor the emergence of collective phenomena.

In a weakly coupled quark–gluon plasma, one can identify a hierarchy of scales that are proportional to the temperature, multiplied by various powers of g. Although these scales may not be well separated when the coupling constant is not sufficiently small, the physical description based on this hierarchy is physically consistent. This is therefore a useful organization principle.

The existence of such a hierarchy can be understood simply, by realizing that the expansion parameter in a field theory at finite temperature depends not only on the strength of the coupling, but also on the magnitude of the relevant thermal fluctuations. Consider for instance a scalar field theory, with a $g^2\phi^4$ interaction. The thermal fluctuations are given by

$$\langle\phi^2\rangle = \int \frac{d^3k}{(2\pi)^3}\frac{n_k}{k}, \qquad n_k = \frac{1}{\mathrm{e}^{k/T}-1}. \tag{1}$$

The integral in (1) is dominated by the largest values of k. We estimate it with an upper cut-off κ and refer to the corresponding value as to "the contribution of the fluctuations at scale κ", and denote it by $\langle\phi^2\rangle_\kappa$. In the same spirit, we approximate the kinetic energy as $\langle(\partial\phi)^2\rangle_\kappa \approx \kappa^2\langle\phi^2\rangle_\kappa$. Taking furthermore $\langle\phi^4\rangle_\kappa \approx \langle\phi^2\rangle_\kappa^2$, one gets as expansion parameter (ratio of potential to kinetic energies)

$$\gamma_\kappa = \frac{g^2\langle\phi^2\rangle_\kappa}{\kappa^2} \simeq \frac{g^2 T}{\kappa}, \qquad \langle\phi^2\rangle_T \sim \kappa T, \tag{2}$$

where the last equality is valid for $\kappa \lesssim T$ (then $n_k \sim T/k$). The fluctuations that dominate the energy density at weak coupling correspond to the plasma particles and have momenta $k \sim T$. For these "hard" fluctuations, $\kappa \sim T$, $\langle\phi^2\rangle_T \sim T^2$, and $\gamma_T \sim g^2$. At this scale, perturbation theory works as well as at zero temperature (with expansion parameter $\sim g^2$, or rather $\alpha = g^2/4\pi$).

The next "natural" scale, commonly referred to as the "soft scale", corresponds to $\kappa \sim gT$, for which $\gamma_{gT} \sim g$. The expansion parameter remains small if g is small, but perturbation theory (for the soft modes) is now an expansion in powers of g rather than g^2: it is therefore less rapidly "convergent". Another phenomenon occurs at the scale gT, important for the present discussion. While the expansion parameter γ_{gT} that controls the self-interactions of the soft fluctuations is small, the coupling between the soft modes and the thermal fluctuations at scale T is not: indeed