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Mapping between the classical and pseudoclassical models of a relativistic spinning particle in external bosonic and fermionic fields. II

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Abstract

The exact solution of a system of bilinear identities derived in the first part of our work [1] for the case of real Grassmann-odd tensor aggregate of the type $(S, V_\mu, {}^*T_{\mu\nu}, A_\mu, P)$ is obtained. The consistency of the solution with a corresponding system of bilinear identities including both the tensor variables and their derivatives $(\dot{S}, \dot{V}_\mu, {}^*\dot{T}_{\mu\nu}, \dot{A}_\mu, \dot{P})$ is considered. The alternative approach in solving of the algebraic system based on introducing complex tensor quantities is discussed. This solution is used in constructing the mapping of the interaction terms of spinning particle with a background (Majorana) fermion field $\Psi^i_{\mathrm{M}\alpha}(x)$. A way of the extension of the obtained results for the case of the Dirac spinors $(\psi_{\mathrm{D}\alpha}, \theta_{\mathrm{D}\alpha})$ and a background Dirac field $\Psi^i_{\mathrm{D}\alpha}(x)$, is suggested. It is shown that for the construction of one-to-one correspondence between the most general spinors and the tensor variables, we need a four-fold increase of the number of the tensor ones. A connection with the higher-order derivative Lagrangians for a point particle and in particular, with the Lagrangian suggested by A.M. Polyakov, is proposed.

Keywords: External fermion field; (Anti)self-dual two-forms; Tetrad; Dirac spinors; Higher-order derivative Lagrangian

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1. Introduction

In the second part of our work we proceed with our analysis of the problem of constructing a mapping between two Lagrangian descriptions of the spin degrees of freedom of a color spinning massive particle interacting with background non-Abelian gauge and fermion fields, started in [1] (to be referred to as "Paper I" throughout this text). Here, we will confine our attention to the interaction of the particle with background fermion field.

In our considerations in Paper I so far we have dealt only with mapping bilinear combinations in the form $\bar{\psi}\hat{O}\psi$, where $\hat{O}$ is a certain (differential) operator or matrix, to the quadratic combinations $\xi_\mu\xi_\nu$, $\xi_\mu\dot{\xi}^\mu$ etc. In our papers [2–4] the simplest model classical Lagrangian of the interaction of a color spinning particle with an external non-Abelian fermion field $\Psi^i_\alpha(x)$ has been suggested. This Lagrangian has the following form:

$$L_\Psi = -eg\hbar(\bar{\theta}\theta)\sqrt{\frac{m}{2}}\Big\{\theta^{\dagger i}\big(\bar{\psi}_\alpha\Psi^i_\alpha(x)\big) + \big(\bar{\Psi}^i_\alpha(x)\psi_\alpha\big)\theta^i\Big\}$$
$$+ eg\hbar(\bar{\theta}\theta)\sqrt{\frac{m}{2}}\left(\frac{C_F}{2T_F}\right)Q^a\Big\{\theta^{\dagger j}(t^a)^{ji}\big(\bar{\psi}_\alpha\Psi^i_\alpha(x)\big) + \big(\bar{\Psi}^i_\alpha(x)\psi_\alpha\big)(t^a)^{ij}\theta^j\Big\}. \tag{1.1}$$

Here, in contrast to [2], we have separated out in an explicit form the dependence of $\hbar$, introduced the dimensional[1] factor $m^{1/2}$ and dimensionless nilpotent one $(\bar{\theta}\theta) \equiv \bar{\theta}_\alpha\theta_\alpha$. It is easy to see that the Lagrangian has the proper dimension for the canonical dimension of an external fermion field (in units $c = 1$)

$$\big[\Psi^i_\alpha(x)\big] \sim \frac{1}{[\,\text{time}\,]^{3/2}}.$$

The commuting spinor $\psi_\alpha(\tau)$ enters linearly into the expression (1.1). This spinor is connected with a set of the anticommuting tensor quantities $(S, V_\mu, {}^*T_{\mu\nu}, \ldots)$ by means of the general relation (I.2.1) (references to formulas in [1] are prefixed by the roman number I), if we preliminarily contract the latter with the auxiliary spinor θ_β

$$\hbar^{1/2}(\bar{\theta}\theta)\psi_\alpha = \frac{1}{4}\Big\{-iS\theta_\alpha + V_\mu(\gamma^\mu\theta)_\alpha$$
$$- \frac{i}{2}{}^*T_{\mu\nu}(\sigma^{\mu\nu}\gamma_5\theta)_\alpha + iA_\mu(\gamma^\mu\gamma_5\theta)_\alpha + P(\gamma_5\theta)_\alpha\Big\}. \tag{1.2}$$

Such an approach of recovering a spinor from the Clifford algebra aggregate by means of an arbitrary auxiliary spinor (so-called the *inverse theorem*) was considered in the commutative case in a number of papers: Zhelnorovich [5], Crawford [6], Keller and Rodriguez-Romo [7], Rodriguez-Romo [8], Lounesto [9] (see also Klauder [10]). If we substitute the representation (1.2) into the Lagrangian (1.1), then in contrast to the mapping of the above-mentioned bilinear combinations, the auxiliary spinor θ_α will already enter explicitly into the Lagrangian L_Ψ as an independent entity. Here, a difficult and subtle question of the dependence of the mapped Lagrangian (1.1) on a concrete choice of θ_α arises, whether it is possible to give a physical meaning of the auxiliary spinor. A similar problem was discussed in paper [5].

Furthermore, in the general expansion (1.2) not all of the functions $(S, V_\mu, {}^*T_{\mu\nu}, \ldots)$ are independent by virtue of the relations (I.C.1)–(I.C.15). Although we have already used some of these

[1] In [2] the commutative spinor ψ_α was considered as a dimensional quantity with the dimension $[\text{mass}]^{1/2}$. In the previous and present works the ψ_α spinor is dimensionless.