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Gluon transport equation with effective mass and dynamical onset of Bose–Einstein condensation

Jean-Paul Blaizot [a], Yin Jiang [b], Jinfeng Liao [b,c,*]

[a] *Institut de Physique Théorique (IPhT), CNRS/URA2306, CEA Saclay, F-91191 Gif-sur-Yvette, France*

[b] *Physics Department and Center for Exploration of Energy and Matter, Indiana University, 2401 N Milo B. Sampson Lane, Bloomington, IN 47408, USA*

[c] *RIKEN BNL Research Center, Bldg. 510A, Brookhaven National Laboratory, Upton, NY 11973, USA*

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Abstract

We study the transport equation describing a dense system of gluons, in the small scattering angle approximation, taking into account medium-generated effective masses of the gluons. We focus on the case of overpopulated systems that are driven to Bose–Einstein condensation on their way to thermalization. The presence of a mass modifies the dispersion relation of the gluon, as compared to the massless case, but it is shown that this does not change qualitatively the scaling behavior in the vicinity of the onset.

Keywords: Quark–gluon plasma; Heavy ion collision; Thermalization; Glasma; Bose–Einstein condensation

1. Introduction

In previous papers [1,2] it was argued that a dense system of gluons, such as those created in the early stages of an ultra-relativistic heavy ion collision, could be driven to Bose–Einstein condensation, as the system evolves towards thermal equilibrium. This was inferred from a detailed study of the kinetic equation that takes into account 2 to 2 scattering, in the small scattering angle approximation. Overpopulation means that the dimensionless number $n/\epsilon^{3/4}$, where n is

* Corresponding author.
E-mail address: liaoji@indiana.edu (J. Liao).

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the number density and ϵ the energy density, exceeds its value in equilibrium. An overpopulated system has too many gluons, relative to its total energy, to be accommodated in a Bose–Einstein distribution, and thermal equilibrium requires the formation of a condensate.

Of course, such a condensate will develop provided the approach to thermal equilibrium proceeds with conservation of both energy and particle number. While energy is certainly conserved, inelastic processes of various kinds may change the number of gluons (see e.g. [3,4]),[1] eventually preventing the formation of a condensate in the true equilibrium state. However, particle number may be approximately conserved during much of the evolution, and this could be enough to approach condensation. Indeed, transport calculations indicate that the amplification of soft modes is a very rapid process, that a chemical potential is indeed dynamically generated and that the onset for condensation can be reached on short time scales. This is confirmed by calculations using the small angle approximation [2], as well as complete solution of the Boltzmann equation [5,7]. There are also indications that inelastic processes could accelerate the amplification of soft modes [6], while the authors of Refs. [8,9] seemingly reach a different conclusion.

Clearly the analysis of inelastic scatterings requires further work, but this is beyond the scope of this paper. Motivation for studying the possibility for gluons to condense comes of course from the desire to better understand how matter produced in a high energy nucleus–nucleus collision evolves towards local thermal equilibrium (see [10,11] for a recent review). But, as we already emphasized in [2], the general issue of the dynamical formation of a condensate is an interesting problem in itself. It is of relevance in the context of cosmology (see e.g. [12]), or cold atom physics (see e.g. [13]). It has be studied using kinetic theory, or classical field simulations (see e.g. [14,15]). In the context of Quantum Chromodynamics, the nature of the condensate remains an interesting, but unsolved, question (for a recent study in a related context, see [16]).

Our goal in this paper is to pursue our general study of the phenomenon within kinetic theory, using a transport equation that incorporates properly the effects of Bose statistics. The fact that the interactions are long range interactions validate the use of the small angle approximation which reduces the transport equation to a Fokker–Planck equation, much easier to solve than the Boltzmann equation, thereby providing more direct analytical insight. This paper, as well as a companion paper, addresses issues that were not discussed in Ref. [2] which is limited to the study of the onset of condensation. We want to extend our work so as to be able to obtain a complete dynamical description of the approach to equilibrium including the formation of a condensate. In order to do so, we need to attribute finite masses to the gluons. Such masses are automatically generated by the coupling to thermal fluctuations, and the proper transport equations that incorporate such self-energy corrections could be derived from first principles. However, for the purpose of the present study, it is sufficient to just give the gluons a mass, and correct appropriately the scattering matrix element. In fact two masses will be introduced. The screening mass m_D regulates the infrared behavior of the collision kernel. The other mass, m, modifies the dispersion relation, and one of the issues that we want to study is how this modification changes the onset of Bose–Einstein condensation. We shall see that, in fact, qualitatively it does not. Finally, the role of the mass m is to allow a clear definition of the equations that describe the evolution of the system beyond the onset, that is, in the presence of the condensate. This will be discussed in a companion paper [17].

[1] Note that quark production, although it decreases the number of gluons, does not necessarily hinds the formation of a condensate [4].