Available online at www.sciencedirect.com
ScienceDirect
Nuclear Physics A 933 (2015) 154–164
NUCLEAR PHYSICS A
www.elsevier.com/locate/nuclphysa

Principal component analysis within nuclear structure

A. Al-Sayed [a,b,*]

[a] *Physics Department, Faculty of Science, Zagazig University, Zagazig, Egypt*
[b] *Physics Department, College of Sciences and Arts in Muthnib, Qassim University, Saudi Arabia*

Received 15 July 2014; received in revised form 3 November 2014; accepted 6 November 2014

Available online 13 November 2014

Abstract

The principal component analysis (PCA) of different parameters affecting collectivity of nuclei predicted to be candidate of the interacting boson model dynamical symmetries is performed. The results show that the use of PCA within nuclear structure can give us a simple way to identify collectivity together with the parameters simultaneously affecting it.

Keywords: Nuclear structure; Interacting boson model; Principal component analysis

1. Introduction

There are two main problems facing nuclear physicists; the lack of knowledge of the exact nature of nuclear force, and the variety of nuclear structures. Many trials are developed to classify the collective behavior of atomic nuclei especially the even–even ones. The interacting boson model (IBM) [1] is a good candidate on this way.

The IBM introduced what is known as dynamical symmetries, which are usually referred to the $U(5)$, $O(6)$ and $SU(3)$ limits. These limits correspond to the familiar spherical, γ-unstable, and well-deformed nuclei respectively. Several trials have been considered to introduce an order parameter to identify nuclei belonging to each dynamical symmetry. Among these is the ratio $R_{4/2}$ [2,3] of excitation energies of the first 4^+ and the first 2^+ excited states. The IBM

* Correspondence to: Physics Department, Faculty of Science, Zagazig University, Zagazig, Egypt.
E-mail address: a.alsayed@zu.edu.eg.

http://dx.doi.org/10.1016/j.nuclphysa.2014.11.003

calculation of energy levels yields values of $R_{4/2} = 2.00, 2.50,$ and 3.33 for the dynamical symmetries $U(5)$, $O(6)$, and $SU(3)$, respectively.

Beside the $R_{4/2}$ ratio, numerous different variables are proposed to facilitate the identification of collective behavior through atomic nuclei. Among these variables; number of protons (Z), quadrupole deformation parameter (β_2) [4], P-factor [5–7], and reduced electric quadrupole transition rate from 0^+ ground state to the first excited 2^+ state $B(E2; 0_1^+ \to 2_1^+)$ or simply $B(E2)\uparrow$ [8].

Most studies focused on classifying nuclei according to a certain variable to facilitate the conclusions. The question is now, what is the effect if we take into consideration all these variables on nuclear structure simultaneously? In other words, do even–even nuclei share the same degree of correlation with these variables? To answer this question, we turn to multivariate analysis in standard statistics.

We describe the data set in Section 2. A brief account of the method of analysis is given in Section 3. In Section 4 results and discussions are presented. The conclusion of this work is outlined in Section 5.

2. Data set

Even–even nuclei are chosen for their abundance and simplicity to be described by nuclear models. The data set is taken from the National Nuclear Data Center [9] to calculate the $R_{4/2}$ ratio, while β_2 data from [10], and $B(E2)\uparrow$ values from a recent update [11] in e^2b^2 units. Nuclei having adopted $B(E2)\uparrow$ are only taken into consideration. We restrict our study to nuclei predicted by the $R_{4/2}$ ratio to represent dynamical symmetries of IBM. We put this condition to simplify the biplot chart as given in the results.

Practically, the $R_{4/2}$ is not a good "order parameter" to distinguish varieties within nuclear structure. The $R_{4/2} = 2.50$ does not correspond only to γ-unstable nuclei, instead, it corresponds to different structures spanning between $O(6)$ to critical point of a spherical to axially deformed transition, $U(5)$–$SU(3)$. Consequently, we will exclude that value 2.5 of the ratio $R_{4/2}$ from our calculations. The range of $2.05 \le R_{4/2} < 2.30$ will be taken into consideration through the current study since most empirically anharmonic vibrational nuclei found within that ratio [12,13]. In this way, we get 10 nuclei in the range $1.95 \le R_{4/2} < 2.05$, 47 nuclei in the range $2.05 \le R_{4/2} < 2.30$, and 41 nuclei of $R_{4/2} \ge 3.25$. The data set is shown in Table 1.

3. Method of analysis

Principal Component Analysis (PCA) is a powerful and widely used technique to analyze data of multivariate structure [14–17]. PCA analyzes a data table representing observations (nuclei) described by several dependent variables, which are, in general, inter-correlated. The goals of PCA are [18] to (a) extract the most important information from the data set, (b) compress the size of the data set by keeping only this important information, (c) simplify the description of the data set, and (d) analyze the structure of the observations and the variables. In order to achieve these goals, PCA computes new variables called *principal components* which are obtained as linear combinations of the original variables. The values of these new variables for the observations are called *factor scores*, and these factor scores can be interpreted geometrically as the projections of the observations onto the principal components [18].

The first step of PCA calculates the covariance matrix, but in the current study the variables ($R_{4/2}$, Z, β_2, P, 2^+, and $B(E2)$) have different variances and units of measurements. Con-