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Quantum design using a multiple internal reflections method in a study of fusion processes in the capture of alpha-particles by nuclei

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Abstract

A high precision method to determine fusion in the capture of α-particles by nuclei is presented. For α-capture by ^{40}Ca and ^{44}Ca, such an approach gives (1) the parameters of the α–nucleus potential and (2) fusion probabilities. This method found new parametrization and fusion probabilities and decreased the error by 41.72 times for $\alpha + {}^{40}$Ca and 34.06 times for $\alpha + {}^{44}$Ca in a description of experimental data in comparison with existing results. We show that the sharp angular momentum cutoff proposed by Glas and Mosel is a rough approximation, Wong's formula and the Hill–Wheeler approach determine the penetrability of the barrier without a correct consideration of the barrier shape, and the WKB approach gives reduced fusion probabilities. Based on our fusion probability formula, we explain the difference between experimental cross-sections for $\alpha + {}^{40}$Ca and $\alpha + {}^{44}$Ca, which is connected with the theory of coexistence of the spherical and deformed shapes in the ground state for nuclei near the neutron magic shell $N = 20$. To provide deeper insight into the physics of nuclei with the new magic number $N = 26$, the cross-section for $\alpha + {}^{46}$Ca is predicted for future experimental tests. The role of nuclear deformations in calculations of the fusion probabilities is analyzed.

Keywords: Alpha-capture; Tunneling; Multiple internal reflections; Fusion probabilities; Sharp angular momentum cut off; Alpha-decay

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1. Introduction

Understanding fusion in nuclear reactions is a fundamental problem of physics [1]. A topic where such processes are important is the synthesis of superheavy nuclei [2,3]. Here, fusion has a crucial role, but its analysis is complicated. Deep insight into the physics of fusion can be obtained from the capture of the α-particles by the nuclei.

Information about fusion in α-capture forms our understanding of interactions between the α-particles and nuclei at distances, where an important role is attributed to the formation of the nucleus from two colliding nuclear objects inside the spatial nuclear region. α–nucleus interactions have been studied most intensively in the context of the α-decay of nuclei (see experimental information [4–16], various microscopic models [17–30], macroscopic cluster models [31–44], fission models [3,46]), and the scattering of the α-particles off nuclei [47–51]. The physics of the fusion processes during α-capture has been investigated less deeply [36–39]. Evaluations of the α-particle capture rates show that they play an important role in nuclear reactions in stars [45,50,52].

A prevailing approach for the determination of the capture cross-sections of the α-particle by the nucleus is based on calculations of the penetrability of the potential barriers, where information about the fusion is not included. While many approaches have been developed for the calculation of the penetrability, there is no generally accepted method to estimate the fusion. Experimentally, these reactions have not been studied deeply: we have the cross-sections for capture of the α-particles by nuclei ^{40}Ca, ^{44}Ca [53], ^{59}Co [54], ^{208}Pb [55], and ^{209}Be [55].

We shall be interesting in information about fusion that can be extracted from the available experimental data. Thus, the aim of this paper is to construct a new approach for obtaining such information. The idea of such an approach can be similar to the inverse theory of scattering [56]. Eberhard et al. proposed a relation that gives information about fusion in the α-capture and compares calculated cross-sections with experimental data at selected energies [53]. Analyzing the existed experimental data, we observe an interesting difference in fusion between α-captures by the ^{40}Ca and ^{44}Ca nuclei, while the inclusion of the other nuclei into the analysis does not result in principally new questions.

A crucial point in such a task is the determination of the barrier penetrability. It turns out that even small improvements in the Wentzel–Kramers–Brillouin (WKB) formula of the penetrability require the reconsideration of the connection between boundary conditions and even the inclusion of the initial (or final) condition into the stationary picture of the studied process. After such a modification, the penetrability becomes more sensitive to the shape of the barrier, and it is already dependent on the potential outside the tunneling region. A fully quantum realization of the approach leads to the essential influence of the calculated penetrability on additional parameters (see [57] for details, proofs, demonstrations). The role of such additional parameters in the determination of the penetrability is larger in some orders in comparison with changes of the resulting probability as a result of any variations in the nuclear deformations and parameters of the interacting potentials if the penetrability is not calculated in the fully quantum approach. In this paper, we shall therefore restrict ourselves to the spherically symmetric α–nucleus potentials. However, the role of the nuclear deformations in the determination of the fusion probabilities in the α-capture task is analyzed in Appendix F.

We generalize the method of multiple internal reflections (MIR method, see [57–62]) to describe the capture of the α-particles by nuclei. We analyze the *sharp angular momentum cutoff approach* previously introduced by Glas and Mosel in [63,64], which has been widely used