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The decay widths, the decay constants, and the branching fractions of a resonant state

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Abstract

We introduce the differential and the total decay widths of a resonant (Gamow) state decaying into a continuum of stable states. When the resonance has several decay modes, we introduce the corresponding partial decay widths and branching fractions. In the approximation that the resonance is sharp, the expressions for the differential, partial and total decay widths of a resonant state bear a close resemblance with the Golden Rule. In such approximation, the branching fractions of a resonant state are the same as the standard branching fractions obtained by way of the Golden Rule. We also introduce dimensionless decay constants along with their associated differential decay constants, and we express experimentally measurable quantities such as the branching fractions and the energy distributions of decay events in terms of those dimensionless decay constants.

Keywords: Decay constant; Branching fractions; Resonant states; Gamow states; Resonances; Golden Rule

1. Introduction

The standard Golden Rule allows us to calculate the transition rate (probability of transition per unit time) from an energy eigenstate of a quantum system into a continuum of energy eigenstates. If $|E_{\rm i}\rangle$ is the eigenstate of an unperturbed Hamiltonian H_0, and if such state is coupled to a state $|E_{\rm f}\rangle$ by a perturbation V, the transition probability per unit of time from the initial state $|E_{\rm i}\rangle$ to the final state $|E_{\rm f}\rangle$ is given, to first order in the perturbation, by

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$$R_{\mathrm{i}\to\mathrm{f}} = \frac{2\pi}{\hbar} \, |\langle E_{\mathrm{f}}|V|E_{\mathrm{i}}\rangle|^2 \, \delta(E_{\mathrm{i}} - E_{\mathrm{f}}) \,, \tag{1.1}$$

where $\langle E_{\mathrm{f}}|V|E_{\mathrm{i}}\rangle$ is the matrix element of the perturbation V between the final and initial states. If the initial state is coupled to a continuum of final states $|E_{\mathrm{f}}\rangle$, and if the density of final states (number of states per unit of energy) is $\rho(E_{\mathrm{f}})$, the transition probability per unit of time from the state $|E_{\mathrm{i}}\rangle$ to the continuum of final states $|E_{\mathrm{f}}\rangle$ is given by

$$\overline{R}_{\mathrm{i}\to\mathrm{f}} = \int \mathrm{d}E_{\mathrm{f}} \, \rho(E_{\mathrm{f}}) R_{\mathrm{i}\to\mathrm{f}} = \frac{2\pi}{\hbar} \, |\langle E_{\mathrm{f}}|V|E_{\mathrm{i}}\rangle|^2 \, \rho(E_{\mathrm{i}}) \,, \tag{1.2}$$

where it has been assumed that the matrix element $\langle E_{\mathrm{f}}|V|E_{\mathrm{i}}\rangle$ is the same for all the states in the continuum. The decay width from the initial state $|E_{\mathrm{i}}\rangle$ to the final state $|E_{\mathrm{f}}\rangle$ is given by

$$\Gamma_{\mathrm{i}\to\mathrm{f}} = \hbar R_{\mathrm{i}\to\mathrm{f}} = 2\pi \, |\langle E_{\mathrm{f}}|V|E_{\mathrm{i}}\rangle|^2 \, \delta(E_{\mathrm{i}} - E_{\mathrm{f}}) \,. \tag{1.3}$$

The decay width from the initial state $|E_{\mathrm{i}}\rangle$ to a continuum of final states $|E_{\mathrm{f}}\rangle$ is given by

$$\overline{\Gamma}_{\mathrm{i}\to\mathrm{f}} = \hbar \overline{R}_{\mathrm{i}\to\mathrm{f}} = 2\pi \, |\langle E_{\mathrm{f}}|V|E_{\mathrm{i}}\rangle|^2 \, \rho(E_{\mathrm{i}}) \,. \tag{1.4}$$

The standard derivation of Eqs. (1.1)–(1.4) is the result of first-order perturbation theory, and it is valid when the initial and final states are stable. The purpose of the present paper is to introduce the analog of Eqs. (1.1)–(1.4) under the assumption that the initial state is described by a resonant (Gamow) state, and that such state decays into a continuum of stable, scattering states.

In Section 2, we introduce the differential and the total decay widths of a resonant state that has only one decay mode. In Section 3, we derive an expression for such decay widths in terms of a truncated Breit–Wigner (Lorentzian) lineshape and the matrix element of the interaction. We also show that, when the resonance is sharp and far from the energy threshold, the expressions for the resonant decay widths bear a strong resemblance with Fermi's Golden Rule. In Section 4, we apply the results of Section 3 to the delta-shell potential. In Section 5, we introduce the partial decay widths of a resonant state that has more than one decay mode, we define the branching fractions for each decay mode, and we point out that, at least in principle, such branching fractions afford a way to falsify the formalism of the present paper. In Section 6, we introduce dimensionless partial and total decay constants along with their associated differential decay constants, we express the branching fractions in terms of them, and we argue that the differential decay constant corresponds to a measurement of the energy distributions of decay events (the invariant mass distributions of particle physics). Section 7 contains our conclusions.

2. Preliminaries

Let $H = H_0 + V$ be a Hamiltonian that produces resonances, where H_0 is the free Hamiltonian and V is the interaction potential. The continuum spectra of both H and H_0 will be assumed to be $[0, \infty)$, as it often occurs in non-relativistic quantum mechanics. For simplicity, we will assume that the continuum spectra are non-degenerate, and that the eigenstates of H_0 can be determined by a single quantum number, the energy E. The eigenstates of the free Hamiltonian will be denoted by $|E\rangle$,

$$H_0|E\rangle = E|E\rangle \,. \tag{2.1}$$

We will denote the resonant state by $|z_{\mathrm{R}}\rangle$,