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Photon– and pion–nucleon interactions in a unitary and causal effective field theory based on the chiral Lagrangian

A. Gasparyan [a,b], M.F.M. Lutz [a,*]

[a] *Gesellschaft für Schwerionenforschung (GSI), Planck Str. 1, 64291 Darmstadt, Germany*
[b] *Institute for Theoretical and Experimental Physics, 117259, B. Cheremushkinskaya 25, Moscow, Russia*

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Abstract

We present and apply a novel scheme for studying photon– and pion–nucleon scattering beyond the threshold region. Partial-wave amplitudes for the γN and πN states are obtained by an analytic extrapolation of subthreshold reaction amplitudes computed in chiral perturbation theory, where the constraints set by electromagnetic-gauge invariance, causality and unitarity are used to stabilize the extrapolation. Based on the chiral Lagrangian we recover the empirical s- and p-wave amplitudes up to energies $\sqrt{s} \simeq 1300$ MeV in terms of the parameters relevant at order Q^3.

Keywords: Chiral symmetry; Unitarity; Causality; Effective field theory; Pion; Nucleon; Photon

1. Introduction

The study of photon– and pion–nucleon interactions has a long history in hadron physics. In recent years such reactions have been successfully used as a quantitative challenge of chiral perturbation theory (χPT), which is a systematic tool to learn about low-energy QCD dynamics [1–3]. The application of χPT is limited to the near threshold region. The pion–nucleon phase shifts have been analyzed in great depth at subleading orders in the chiral expansion [4–6]. Pion photoproduction was studied in [7–10]. Compton scattering was considered in [1,11].

* Corresponding author.
E-mail address: m.lutz@gsi.de (M.F.M. Lutz).

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Though there are many model computations (see e.g. [12–18]) that address such reactions at energies significantly larger than threshold it is an open challenge to further develop systematic effective field theories that have a larger applicability domain than χPT and are predictive nevertheless. The inclusion of the isobar as an explicit degree of freedom in the chiral Lagrangian is investigated in [19–27]. This leads to an extension of the applicability domain of the chiral Lagrangian which is based on power counting rules. A successful description of scattering data in the isobar region requires the systematic summation of an infinite number of terms. Such a summation may be motivated by generalized counting rules [24] to be applied directly to the S-matrix. Alternatively, the required summation may be justified by the request that the process is described in accordance with the unitarity constraint [22,28–31]. The counting rules are applied to irreducible diagrams only. An infinite number of reducible diagrams being summed by the unitarity request [22]. This is analogous to the scheme proposed by Weinberg for the nucleon–nucleon scattering problem [32,33].

The purpose of this work is to develop a unified description of photon and pion scattering off the nucleon based on the chiral Lagrangian. We aim at a description from threshold up to and beyond the isobar region in terms of partial-wave amplitudes that are consistent with the constraints set by causality and unitarity. Our analysis is based on the chiral Lagrangian with pion and nucleon fields truncated at order Q^3. We do not consider an explicit isobar field in the chiral Lagrangian. The physics of the isobar resonance enters our scheme by an infinite summation of higher order counter terms in the chiral Lagrangian. The particular summation is performed in accordance with unitarity and causality.

Our work is based on a scheme proposed in [34]. We develop a suitable extension to be applied to pion–nucleon scattering, pion photoproduction and Compton scattering. The scheme is based on an analytic extrapolation of subthreshold scattering amplitudes that is controlled by constraints set by electromagnetic-gauge invariance, causality and unitarity. Unitarized scattering amplitudes are obtained which have left-hand cut structures in accordance with causality. The latter are solutions of non-linear integral equations that are solved by N/D techniques. The integral equations are imposed on partial-wave amplitudes that are free of kinematical zeros and singularities. Such amplitudes are constructed in Appendix A. An essential ingredient of the scheme is the analytic continuation of the generalized potentials that determine the partial-wave amplitudes via the non-linear integral equation. We discuss the analytic structure of the generalized potentials in detail and construct suitable conformal mappings in terms of which the analytic continuation is performed systematically. Contributions from far distant left-hand cut structures are represented by power series in the conformal variables.

The relevant counter terms of the Lagrangian are adjusted to the empirical data available for photon and pion scattering off the nucleon. We focus on the s- and p-wave partial-wave amplitudes and do not consider inelastic channels with two or more pions. We recover the empirical s- and p-wave pion–nucleon phase shifts up to about 1300 MeV quantitatively. The pion photoproduction process is analyzed in terms of its multipole decomposition. Given the significant ambiguities in those multipoles we offer a more direct comparison of our results with differential cross sections and polarization data. A quantitative reproduction of the data set up to energies of about $\sqrt{s} \simeq 1300$ MeV is achieved.

2. Analytic extrapolation of subthreshold scattering amplitudes

In this section we perform a systematic analytic continuation of the reactions $\pi N \to \gamma N, \pi N$ and $\gamma N \to \gamma N, \pi N$ based on the chiral Lagrangian [5,2]. After a review of the interaction terms