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Collective electronic pulsation around giant nuclei in the Thomas–Fermi model

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Abstract

Based on the Thomas–Fermi solution for compressed electron gas around a giant nucleus, we study electric pulsations of electron number-density, pressure and electric fields, which could be caused by an external perturbations acting on the nucleus or the electrons themselves. We numerically obtain the eigen-frequencies and eigen-functions for stationary pulsation modes that fulfill the boundary-value problem established by electron-number and energy–momentum conservation, equation of state, and Maxwell's equations, as well as physical boundary conditions, and assume the nucleons in β-equilibrium at nuclear density. We particularly study the configuration of ultra-relativistic electrons with a large fraction contained within the nucleus. Such configurations can be realized for a giant nucleus or high external compression on the electrons. The lowest modes turn out to be heavily influenced by the relativistic plasma frequency induced by the positive charge background in the nucleus. Our results can be applied to heavy nuclei in the neutron star crust, as well as to the whole core of a neutron star. We discuss the possibility to apply our results to dynamic nuclei using the spectral method.

Keywords: Thomas–Fermi model; Electromagnetic pulsation; Giant atom; Neutron star

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1. Introduction

The Thomas–Fermi model that was found independently by Thomas [1] and Fermi [2] in 1927 quantitatively describes neutral and ionized atoms of large electron-numbers with great success (see for example Refs. [3–11]). The Thomas–Fermi solution turns out to be exact when the electron-number goes to infinity [12]. Essentially, the Thomas–Fermi model is a semi-classical and mean-field approach to the problem of many electrons around a nucleus with a large number of protons. It describes a neutral or charged static equilibrium configuration of electrons around a nucleus with or without compression. While it turned out to be of limited use in the realm of atomic physics, it has been applied very successfully in astrophysical settings (see for example Refs. [13–17]).

In this article, on the basis of the Thomas–Fermi solution, namely the equilibrium configurations of electrons compressed around a giant nucleus, we investigate radial perturbations (electric pulsations) with spherical symmetry upon the equilibrium configurations. We find that the spectrum of pulsation modes is determined by two effects: (i) outside the nucleus the speed of sound of the electron gas determines propagation, with possible contributions from both non- and ultra-relativistic zones, while (ii) inside the nucleus there is an additional contribution due to the relativistic plasma frequency induced by the nuclear positive charge background. For sufficiently low frequency modes this leads to the perturbation dying away exponentially within the volume of the nucleus, rendering it effectively unavailable for wave propagation. To study the configuration of ultra-relativistic electrons with a large fraction contained within the nucleus, we choose a proton number $Z = 10^6$ for the purpose of practical numerical simulation and illustration of (ii). While the effects we observe are also present at smaller $Z \approx 10^3$–10^4, a more realistic configuration that might be expected in the very deep crust of neutron stars in the form of *pasta equation of state*, they are less pronounced (see Fig. 5), and high electron densities partially rely on the gravitational pressure in this case. Instead we choose $Z = 10^6$ because here most electrons are kept inside the nucleus solely by the electric interaction, and β-equilibrium is saturated throughout the nucleus. For the effects we observe it is essential that electron densities approach proton densities inside the nucleus, in any other case including high pressure laboratory setups, the spectrum of the vibrational modes would be dominated by the equation of state and the corresponding speed of sound of the electron gas, the only feature being a transition from non- to ultra-relativistic conditions (see discussion in the conclusions and Fig. 5).

The electrons around a static nucleus are treated as a perfect fluid described by thermodynamic number-density n, energy-density ρ and pressure p with non-vanishing electric potential and field. In addition to the equation of state at zero temperature, these physical quantities fully obey the Maxwell field-equation, Euler equation and the first thermodynamical law that follows from electron-number and energy–momentum conservations. This system is completely determined with appropriate physical boundary conditions. In order to study the perturbative electric pulsations, we have linearized these relations and equations, based on the prescription of Eulerian and Lagrangian perturbations of the Thomas–Fermi equilibrium configuration. As a result we obtain a homogeneous second-order differential equation for perturbations satisfying appropriate physical boundary conditions.

As a first step, we focus on the stationary solution ($\propto e^{i\omega t}$) with the characteristic eigenfrequencies ω of electric perturbations (pulsations) of the Thomas–Fermi system, so as to understand what are time-scales (inverse frequencies) at which the system responds to external