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ScienceDirect
Nuclear Physics A 941 (2015) 121–144
NUCLEAR PHYSICS A
www.elsevier.com/locate/nuclphysa

Dynamics of two-cluster systems in phase space

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Received 19 March 2015; received in revised form 9 June 2015; accepted 12 June 2015

Available online 17 June 2015

Abstract

We present a phase-space representation of quantum state vectors for two-cluster systems. Density distributions in the Fock–Bargmann space are constructed for bound and resonance states of ^{6,7}Li and ^{7,8}Be, provided that all these nuclei are treated within a microscopic two-cluster model. The density distribution in the phase space is compared with those in the coordinate and momentum representations. Bound states realize themselves in a compact area of the phase space, as also do narrow resonance states. We establish the quantitative boundaries of this region in the phase space for the nuclei under consideration. Quantum trajectories are demonstrated to approach their classical limit with increasing energy.

Keywords: Phase portrait; Fock–Bargmann space; Coherent state; Resonating group method

1. Introduction

The idea of formulation of quantum mechanics in a phase space is discussed in numerous theoretical papers [1–6]. The majority of such investigations are concentrated on establishing the link between quantum and classical mechanics. Due to the uncertainty principle there is no unique definition of the phase space. By this reason, different quantum phase space distribution has been proposed. In particular, in the phase-space representation of Wigner and Husimi a quantum state is represented by a distribution function (see the definitions, for instance, in [1,2]), and the equations of motion are of the Liouville type.

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http://dx.doi.org/10.1016/j.nuclphysa.2015.06.006
0375-9474/

A state-vector representation is another possibility to describe the dynamics of a quantum system in phase space. In this case a quantum state is represented by a wave function, and the equations of motion are of the Schrödinger type. The definition of the phase-space representation is related to the choice of an operator which should be diagonal in this representation. In the coordinate representation the coordinate operator is diagonal, while the momentum operator is non-local. At the same time, the momentum representation diagonalizes the momentum operator, and the coordinate operator is non-local. Obviously, the coordinate operator and the momentum operator cannot be diagonal simultaneously due to the uncertainty principle. Hence, one should seek for another operator.

In Ref. [3], mention was made that representation of a quantum state as a probability amplitude depending on two real variables related to the coordinate and momentum dates back to the papers of Fock [7] and Bargmann [8]. In the Fock–Bargmann space, a quantum state is represented as an entire function of a complex variable, with real and imaginary part of this variable being proportional to the coordinate and momentum, correspondingly. The Fock–Bargmann representation diagonalizes the creation operator, while the annihilation operator is non-local.

A quantum problem can be resolved in any one of the above-mentioned representations. The Fock–Bargmann image of a wave function can be obtained from the wave function in the coordinate representation by a linear mapping, while the Husimi and the Wigner distribution functions are bilinear with respect to the wave function in the coordinate representation. However, the Fock–Bargmann representation is closely related to the Husimi distribution. The latter distribution is equal to the square of the Fock–Bargmann image of the corresponding wave function multiplied by the Bargmann measure.

In Refs. [4,5], Torres-Vega and Frederic suggested a quantum-state vector phase-space representation. The authors postulated the existence of a complete basis of states $|q, p\rangle$ such that in the phase space a quantum state $|\psi\rangle$ is represented by an $\mathcal{L}^2(2)$ wave function $\psi(p, q) = \langle q, p|\psi\rangle$. Here q, p are real values and the operators of coordinate and momentum in this basis take the form:

$$\widehat{Q} = \frac{q}{2} + \imath\hbar\frac{\partial}{\partial p}, \qquad \widehat{P} = \frac{p}{2} - \imath\hbar\frac{\partial}{\partial q}. \tag{1}$$

Thus Torres-Vega and Frederic developed a wave-function formulation of quantum mechanics in a phase space. Wave functions are governed by the Schrödinger equation in the phase space, while square of absolute value of the wave function plays the role of the probability density in the phase space obeying the Liouville equation. However, the quantum-state vector phase-space representation is not uniquely defined, because there exists an infinite number of bases depending on two real variables p, q which result in the foregoing expression of the coordinate and momentum operators $\widehat{Q}$ and $\widehat{P}$.

In Ref. [5], the authors used coherent states as basis vectors and demonstrated that any coherent state representation leads to expression (1) for the coordinate and momentum operators $\widehat{Q}$ and $\widehat{P}$. Moreover, they concluded that only the coherent state representation makes it possible to define the operators of coordinate and momentum in such a manner.

Following Klauder and Perelomov, in [5] a set of coherent states is defined as a result of action of the Weyl operator $\widehat{D}(q, p)$ (a translation operator in the phase space)

$$\widehat{D}(q, p) = \exp\left\{\frac{i}{\hbar}\left(p\widehat{Q} - q\widehat{P}\right)\right\}$$

to any normalized vector $|\chi\rangle$: