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Mode-by-mode hydrodynamics: Ideas and concepts

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Abstract

The main ideas, technical concepts and perspectives for a mode resolved description of the hydrodynamical regime of relativistic heavy ion collisions are discussed. A background-fluctuation splitting and a Bessel–Fourier expansion for the fluctuating part of the hydrodynamical fields allows for a complete characterization of initial conditions, the fluid dynamical propagation of single modes, the study of interaction effects between modes, the determination of the associated particle spectra and the generalization of the whole program to event-by-event correlations and probability distributions.

Keywords: Relativistic heavy ion collisions; Fluid dynamics

In recent years it has become apparent what one may call the “fluid dynamic standard model of heavy ion collisions” according to which initial density anisotropies are evolved to final momentum anisotropies by almost ideal hydrodynamics works surprisingly well, see [1–4] for recent reviews. Some of the remaining puzzles are about the nature of the initial state directly after the collision and the early out-of-equilibrium dynamics that drives it towards local equilibrium on a rather short time scale, see [5] for an overview over recent literature on that question. On a more quantitative level one would like to better understand how thermodynamic and transport properties govern the hydrodynamic regime in order to allow for precise comparison between theoretical calculations and experimental measurements of these material properties of strongly interacting quantum field theory.

Interesting new insights into both the properties of the initial state and the material properties of QCD in the hydrodynamic regime may come from the study of fluctuations in the hydrodynamic fields. More specific, from many models or on general grounds one expects event-by-event fluctuations around the average in the hydrodynamical fields such as energy density ϵ, fluid

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velocity u^μ, shear stress $\pi^{\mu\nu}$ (and more general also baryon number density n_B, electric charge density, electromagnetic fields and so on) at the initialization time τ_0 where the hydrodynamical description becomes (approximately) valid. These fluctuations should contain interesting information from early times, both from the initial state and the early non-equilibrium dynamics. Their dynamical evolution is governed by universal fluid dynamic equations, depending only on the thermodynamic and transport properties. Since one can distinguish fluctuations of different characteristic spatial size by their wave numbers, the information content is much richer than for averaged quantities only.

There is an interesting analogy to cosmology. Indeed, the cosmic microwave background also contains interesting information from early times and the time evolution is sensitive to the history of the universe. The spectrum of fluctuations involves many numbers (or whole functions) that can be compared between theory and experiment. Historically, the detailed study of this spectrum has lead to quite a detailed quantitative understanding of cosmology and one may hope that a similar progress is possible in heavy ion physics, as well.

So what would be the elements of a theoretical program that aims for a precise understanding of event-by-event hydrodynamical fluctuations in analogy to the corresponding program in cosmology? One may split this into four points:

1. Initial fluctuations at the initialization time of the hydrodynamic description have to be characterized and quantified completely. Ideally this is done in a way that is independent of specific models of the initial state.
2. Fluctuations have to be propagated through the hydrodynamical regime and one would like to know how precisely this evolution is affected by the thermodynamical and transport properties (which are functions of temperature and chemical potential).
3. The impact of the fluctuations in hydrodynamical fields onto the particle spectra generated at freeze-out must be understood and quantified in detail.
4. The effects of fluctuations generated from non-hydro sources (such as, for example, jets) should be quantified and eventually taken into account, as well.

We will go through these points again below. It is clear that they can be implemented in different ways in principle. Numerical event-by-event simulations for specific models of the initial state and specific choices of thermodynamic and transport properties implement already some parts of the above program. However, we will argue that a more analytic approach (based on a mode-decomposition) has certain advantages and in particular it allows to disentangle the different steps above. This will be useful in order to study more systematically what can be learned from experimental results, how they constrain models of the initial state (and early dynamics), how they constrain the thermodynamic and transport properties in the hydrodynamic regime and to what extent the two things are complementary.

Let us first concentrate on the initial state. For a particular event the initial conditions on the initialization surface (usually taken to be the one of fixed Bjorken time $\tau = \sqrt{t^2 - z^2} = \tau_0$) are fixed in terms of a number of independent functions which one may choose to be enthalpy density w, three independent components of the fluid velocity u^μ and five independent components of the shear stress $\pi^{\mu\nu}$. (One may also take non-zero bulk viscous pressure, baryon number density, electric charge density, etc., into account.) One can always split these hydrodynamical fields into a background and a fluctuating part, for example,

$$w = w_{\mathrm{BG}} + \delta w, \qquad u^\mu = u^\mu_{\mathrm{BG}} + \delta u^\mu. \tag{1}$$