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Anomalous transport with overlap fermions

P.V. Buividovich

Institute of Theoretical Physics, University of Regensburg, Universitätsstraße 31, Regensburg D-93053, Germany

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Abstract

Anomalous correlators of vector and axial currents which enter the Kubo formulae for the chiral magnetic and the chiral separation conductivities are explicitly calculated for free overlap fermions on the lattice. The results are confronted with continuum calculations in the finite-temperature regularization, and a subtle point of such regularization for chiral magnetic conductivity related to the correct counting of the chiral states is highlighted. In agreement with some previous claims in the literature, we find that in a lattice regularization which respects gauge invariance, the chiral magnetic conductivity vanishes. We point out that the relation of anomalous transport coefficients to axial anomaly is nontrivial due to the non-commutativity of their infrared limit and the Taylor expansion in baryon or chiral chemical potential. In particular, we argue that the vector and axial Ward identities fix the asymptotic behavior of anomalous current–current correlators in the limit of large momenta. Basing on the work of Knecht et al. on the perturbative non-renormalization of the transverse part of the correlator of two vector and one axial currents, we demonstrate that the relation of the anomalous vector–vector correlator to axial anomaly holds perturbatively in massless QCD but might be subject to non-perturbative corrections. Finally, we identify kinematical regimes in which the anomalous transport coefficients can be extracted from lattice measurements.

Keywords: Chiral magnetic effect; Chiral separation effect; Chiral magnetic conductivity; Axial anomaly

1. Introduction

Anomaly-related transport properties of systems of chiral fermions have become recently a subject of intense theoretical and experimental studies. Anomalous transport phenomena of

E-mail address: pavel.buividovich@physik.uni-regensburg.de.

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particular interest for the physics of heavy-ion collisions are the Chiral Separation Effect (CSE) [1,2] and the Chiral Magnetic Effect (CME) [3]. The Chiral Separation Effect is the generation of an axial current j_i^A along the magnetic field B_i at nonzero chemical potential μ_V:

$$j_i^A = \sigma_{CSE} B_i, \qquad \sigma_{CSE} = \frac{1}{2\pi^2}\mu_V. \tag{1}$$

The Chiral Magnetic Effect is the generation of an electric current j_i^V along the magnetic field at nonzero chiral chemical potential μ_A:

$$j_i^V = \sigma_{CME} B_i, \qquad \sigma_{CME} = \frac{1}{2\pi^2}\mu_A. \tag{2}$$

For simplicity in this paper we assume that the numbers N_c and N_f of fermion colors and flavors and also the fermion charge are all equal to one. These numbers enter all the expressions which we use in this work as simple pre-factors and can be restored, if necessary, in a completely straightforward way.

While the chemical potential μ_V couples to the total charge $q_R + q_L$ of both right- and left-handed fermions, the chiral chemical potential μ_A couples to the difference $q_R - q_L$. Both Chiral Magnetic and Chiral Separation effects combine into a novel type of excitation of a plasma of chiral fermions in an external magnetic field – the Chiral Magnetic Wave [4,5]. All these effects should lead to specific anisotropies and correlations in the distributions of charged particles produced in heavy ion collisions [6–8]. In a recent work [9] first numerical simulations of hydrodynamical evolution in heavy-ion collisions which took into account anomalous transport were performed and some specific experimental signatures of the importance of anomalous transport phenomena were pointed out. Even though the experimental evidence for these effects from heavy-ion collision experiments is somewhat controversial at present (see e.g. [10]), anomalous transport phenomena might also be interesting in other areas of physics, for example, for the recently discovered Weyl semi-metals [11–14].

In contrast to the conventional transport phenomena such as ohmic conductivity or viscous flow, anomalous transport is a property of equilibrium state of quantum systems and does not lead to any dissipation of heat. For this reason they are also ideally suited for lattice studies. Indeed, since within the linear response approximation anomalous transport coefficients can be extracted from static correlators of axial and vector currents and the energy–momentum tensor [15,16], tedious analytic continuation of lattice data from Euclidean to real (Minkowski) time which is typically a source of large uncertainties and systematical errors turns out to be unnecessary.

In the original works [1,2] and [3] (see also [17,18] for some earlier derivations, and [19] for a later work on non-renormalizability of CME) it was argued that the values (1) and (2) of the transport coefficients σ_{CSE} and σ_{CME} (which we will call the chiral separation conductivity and the chiral magnetic conductivity, respectively) are related to the axial anomaly and thus do not receive any corrections due to interactions. Somewhat later it has also been realized that the values of σ_{CSE} and σ_{CME} can be strongly constrained based on purely thermodynamical arguments if one requires that the divergence of an entropy current in "anomalous hydrodynamics" is non-negative [20–26]. Anomalous transport coefficients (1) and (2) are also reproduced in classical kinetic theory, where the quantum anomaly can be incorporated in terms of the nontrivial flux of the momentum-space Berry curvature through the Fermi surface [27,28].

However, some further analytic and numerical studies [29–31] indicated that both the Chiral Magnetic Effect and the Chiral Separation Effect might still receive corrections in interacting field theories. It was recently stressed in [32] that if one couples dynamical gauge fields to the currents entering the anomalous correlators, the anomaly equations are no longer restrictive enough