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Fluctuations in relativistic causal hydrodynamics

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Abstract

Formalism to calculate the hydrodynamic fluctuations by applying the Onsager theory to the relativistic Navier–Stokes equation is already known. In this work, we calculate hydrodynamic fluctuations within the framework of the second order hydrodynamics of Müller, Israel and Stewart and its generalization to the third order. We have also calculated the fluctuations for several other causal hydrodynamical equations. We show that the form for the Onsager-coefficients and form of the correlation functions remain the same as those obtained by the relativistic Navier–Stokes equation and do not depend on any specific model of hydrodynamics. Further we numerically investigate evolution of the correlation function using the one dimensional boost-invariant (Bjorken) flow. We compare the correlation functions obtained using the causal hydrodynamics with the correlation function for the relativistic Navier–Stokes equation. We find that the qualitative behavior of the correlation functions remains the same for all the models of the causal hydrodynamics.

Keywords: Fluctuations; Onsager coefficients; Causal hydrodynamics

1. Introduction

A study of fluctuations in continuous media is of great interest in physics and it can provide a link between the macroscopic and microscopic points of view. A macroscopic theory such as hydrodynamics provides a simplest possible description of a complicated many-body system in terms of space–time evolution of the mean or averaged quantities like energy density, pressure,

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flow velocity, etc. The fluctuation theory studies small deviations from the mean behavior and it can help in calculating correlation functions for the macroscopic variables [1,2]. In context of relativistic hydrodynamics, results of the fluctuation–dissipation theorem have been studied in Refs. [3,4]. In Ref. [3] the authors have studied the fluctuation in the contexts of general-relativistic Navier–Stokes theory. A more general framework of hydrodynamics described as the divergence type theory (DTT) [5] was considered in Ref. [4]. It ought to be noted that recently in an interesting work in Ref. [6], the authors have applied results of the fluctuation–dissipation theorem to the relativistic *Navier–Stokes* theory of hydrodynamics and calculated the two particles correlators for the one-dimensional hydrodynamics (Bjorken) flow relevant for the relativistic heavy-ion collision experiments at RHIC and LHC. The authors obtained several analytical results for two particle correlation functions. Further, in Ref. [7], the authors have studied the effect of thermal conductivity on the correlation function using the Bjorken-flow. It should be noted here that it is well-known that relativistic Navier–Stokes theory exhibits acausal behavior which can give rise to unphysical instabilities [8]. However the causality can be restored if the terms with higher orders are included in the hydrodynamics as indicated by the Maxwell–Cattaneo law [9]. Indeed these issues do not arise in the second-order causal hydrodynamics theory developed by Müller, Israel and Stewart (MIS) [10]. Form of the Navier–Stokes equations can be determined from the second law of thermodynamics $\partial_\mu S^\mu \geqslant 0$, where S^μ denotes the equilibrium entropy current. However, in general it is not possible for an out-of-equilibrium fluid to have an equilibrium entropy current [9]. In MIS hydrodynamics out-of-equilibrium current can have contributions from dissipative processes like the effect of viscosity and the heat conduction. This has an interesting analogy with the irreversible thermodynamics [11,12]. Further, the MIS hydrodynamics has been extensively applied to study the relativistic heavy-ion collisions [9,13–15] and also in cosmology [16]. Later this formalism was extended to include the effect of third order terms in the gradient expansion [17]. Recently, it has been shown that the derivation of the MIS equations from the underlying kinetic equation may not be unique, there may exist a more general set of hydrodynamic equations which may allow one to obtain MIS equations as a special case [19,18].

Finally, it should be mentioned here that although the divergent type theory (DTT) of relativistic fluid of Geroch–Lindblom [5] allows for a consistent proof of causality and stability of its solutions, it is far from direct thermodynamic intuition. Moreover, the connection between the DTTs and MIS or other causal hydrodynamics theories is not yet clearly established.

In this work we apply the fluctuation–dissipation theorem to MIS equations and also to the hydrodynamics models developed by Denicol, Koide and Rischke (DKR) [19], Jaiswal, Bhalerao and Pal (JBP) [18] and other models based on MIS approach [10,17,20]. Further, we apply these results to study the hydrodynamical evolution using $1+0$ dimensional Bjorken flow. In particular, we calculate the correlators using the Onsager coefficients for various relativistic hydrodynamical theories.

2. Fluctuations and correlations in hydrodynamics

In thermodynamic equilibrium entropy of the system S which is a function of the additive quantities x_k becomes maximum. In equilibrium, S satisfies the condition $X_k = -\frac{\partial S}{\partial x_k} = 0$. However, when the system is slightly away from the equilibrium the generalized forces $X_k \neq 0$ and $\frac{dx_i}{dt} = -\gamma_{ik} X_k + \xi_i$, the summation convention is implied, describes the flux associated with the quantity x_i, here ξ_i are the random forces or the noise term and γ_{ik} are the Onsager coefficients.