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The statistical multifragmentation model for liquid–gas phase transition with a compressible nuclear liquid

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Abstract

We propose a new formulation of the statistical multifragmentation model based on the analysis of the virial expansion for a system of the nuclear fragments of all sizes. The developed model not only allows us to account for short-range repulsion, but also to calculate the surface free energy which is induced by the interaction between the fragments. Also we propose a new parameterization for the liquid phase pressure which allows us to introduce a compressible nuclear liquid into the statistical multifragmentation model. The resulting model is exactly solvable and has no irregular behavior of the isotherms in the mixed phase region that is typical for mean-field models. The general conditions for the 1-st and 2-nd (or higher) order phase transitions are formulated. It is shown that all endpoints of the present model phase diagram are the tricritical points, if the Fisher exponent τ is in the range $\frac{3}{2} \leqslant \tau \leqslant 2$. The treatment of nuclear liquid compressibility allows us to reduce the tricritical endpoint density of the statistical multifragmentation model to one third of the normal nuclear density. A specific attention is paid to the fragment size distributions in the region of a negative surface tension at supercritical temperatures.

Keywords: Statistical multifragmentation model; Van der Waals extrapolation; Surface tension; Compressible nuclear liquid

1. Introduction

More than 30 years the statistical multifragmentation model (SMM) [1] is playing the leading role in studies of the nuclear multifragmentation reactions [2,3] which, probably, is one of

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the most spectacular phenomena that is available for exploration in nuclear reactions at intermediate energies. Additionally, the SMM greatly stimulated the studies of phase transition (PT) in finite systems [2–5] and investigation of non-homogeneous phases of strongly interacting matter in astrophysics [6–8] and heavy-ion collisions [9–11].

A simplified version of the SMM without the Coulomb and asymmetry terms was proposed in [12,13]. Its analytical solution was obtained in [14,15], where an additional parameter, the Fisher exponent τ, was introduced in the model. The value of the Fisher exponent extracted from nuclear experiments [16,17] turned out to be $\tau \simeq 1.8$–1.9, i.e. well below the prediction of the Fisher droplet model (FDM) $\tau_F \simeq 2.209$ [18,19], but in a good correspondence with the critical exponent analysis made in [20] for the simplified SMM. This fact initiated new attempts to analyze the multifragmentation data in order to extract the values of the exponent τ and critical temperature within the original SMM [21–24] and within its simplified versions [25]. Note that these results clearly indicate an existence of effective power law in the data which is not postulated in the standard SMM, but is automatically generated in the final distributions of fragments [21–25]. Also these studies gave a first evidence that the nuclear liquid–gas PT has the tricritical endpoint rather than the critical one [20]. Finally, it is necessary to stress that the experimental searches for the signals of nuclear liquid–gas PT led to a tremendous progress in experimental techniques and theoretical approaches (see e.g. [26–30] and references therein).

Despite the obvious successes of the SMM in describing many sets of experimental data there are two simple conceptual questions: why does the SMM perfectly work in describing the low density nuclear vapor that appears at the freeze-out stage, and why does the SMM predict [14,15] that the critical point of the liquid–gas PT is located at $\rho = \rho_0$ instead of $\rho \simeq \frac{1}{3}\rho_0$. Here $\rho_0 \simeq 0.16\ \mathrm{fm}^3$ is the normal nuclear density which is a maximal density of nuclear liquid in the SMM [14,15]. At first glance the answer on the second question seems to be very trivial: the SMM considers all nuclear fragments as incompressible droplets. This can be seen from the eigen volume of the k-nucleon fragment which is $V_k = \frac{k}{\rho_0}$. Thus, the limiting particle density ρ_0 appears in the SMM due to the Van der Waals like treatment of the short range repulsion between the fragments. However, in this case we really face a difficult problem to get the answer on the first of the above questions and to explain the reason why the SMM is so good in describing the experimental data at freeze-out density which is between $\frac{1}{6}$ and $\frac{1}{3}$ of ρ_0. As well known, to describe the thermodynamics at low densities one has to use the virial expansion and to account for, at least, the second virial coefficients $a_{jk} = \frac{2}{3}\pi(R_j + R_k)^3$ between all pairs of fragments of the hard core radii R_j and R_k [31–34]. The real problem, however, is that the SMM employs not the second virial coefficients which provide the description of low density matter, but uses the eigen volumes of the k-nucleon fragments $V_k = \frac{4}{3}\pi R_k^3$, which, usually, appear in the high density limit [35,36]! In order to understand why the SMM is successful at low densities, we have to return to its basic assumptions and find out, how the virial coefficients appear in this model. In order to demonstrate this idea straightforwardly, we employ the simplified SMM.

The simplified version of the SMM [12,13] which is solved analytically in [14,15] is much more elaborate than the FDM [18], since in contrast to the latter one, the SMM explicitly contains the nonzero proper sizes of all fragments and, hence, the liquid phase. However, in the standard SMM the nuclear liquid is incompressible, that is too rough approximation at higher temperatures. By this reason the critical and tricritical endpoints of the simplified SMM [14,20] appear at the density of a liquid phase $\rho = \rho_l(T = 0) = \rho_0$, while in ordinary substances the critical density is about one third of that one for low temperature liquid phase [31,37,38]. In present paper we modify the simplified SMM to account for the compressibility of nuclear liquid and show how the equation for the surface tension coefficient induced by interaction between the nuclear