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Test of the coherent state approach in the axially deformed region

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Abstract

Coherent State Approach on the Interacting Boson Model (IBM) is tested between the axially deformed and γ-soft transitional regions. Excitation energies of the bands are obtained for the simple IBM Hamiltonian written for this region. Deformation parameter which minimizes the energy of the state is found in a closed form as a function of boson number for each band. Then energy of state within the band is defined by using the moment of inertia obtained from the solution of cranking problem. Matrix elements of quadrupole moment operator and reduced electric quadrupole transitions are given. Then the Coherent State Approach is tested in the case of some selected Hf isotopes.

Keywords: Coherent state approach; Interacting boson model; Transitional nuclei

1. Introduction

Studies on phase transitions in atomic nuclei have been increased following the introduction of critical point symmetries, $E(5)$ and $X(5)$ [1,2]. These critical points correspond to second-order and first-order phase transitions from vibrational to γ-unstable and to prolate-deformed structures, respectively. Within the collective model framework, structural properties of nuclei which are at the critical points can be found analytically from the solution of collective Bohr Hamiltonian [3] with an appropriate potential.

In addition to the collective model, phase transitions can be studied by using different models. One of the most successful models is the Interacting Boson Model (IBM) [4], in which nucleons outside the closed shell are taken into account as a boson and, in the earliest version of the model,

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these bosons carry $\ell = 0\hbar$ (s-bosons) and $\ell = 2\hbar$ (d-bosons) angular momentum. This leads to a description in terms of the unitary group, $U(6)$ [5]. Different reductions of $U(6)$ lead to three dynamical symmetry limits labeled by $U(5)$ [6], $SU(3)$ [7] and $O(6)$ [8], which correspond to spherical vibrator, deformed-rotor and γ-unstable rotor, respectively. In the dynamical symmetry limits, energy eigenvalues are given in terms of eigenvalues of Casimir operators [4], while away from these limits, the exact solution no longer exists and one can easily diagonalize any IBM Hamiltonian numerically.

The IBM is an algebraic model and the geometric discussion of IBM is provided by the intrinsic state formalism [9–12]. The technique of the IBM coherent state formalism to study the intrinsic state and the associated collective properties has been reviewed by Leviatan [13]. Static and dynamic properties of nuclei can be easily determined with considerable accuracy [14]. The validity of the formalism was tested far from the dynamical symmetry limits and it was found that it provides approximations to the exact results which are correct up to the order $1/N$ [15]. The classical limit of the IBM-2 Hamiltonian has been used to study the phase structure of boson system with proton–neutron degrees of freedom [16]. More recently, the method has been developed to construct a Hamiltonian of the IBM by mapping from the potential energy surface of Skyrme energy density functional [17,18] onto the coherent-state expectation value of the IBM Hamiltonian [19,20].

For rotational nuclei, definition of moment of inertia in the IBM has been obtained in a closed form up to the first order and higher order in cranking frequency ω from the solution of self-consistent cranking problem [21,22]. The concept of rotational cranking in the IBM has been used to study the general feature of the quantum phase transition in finite nuclei with non-zero angular frequency [23,24]. The same technique has been implemented in the fermion–boson mapping for rotational nuclei [25]. On the other hand, the Hf isotopes and some adjacent isotopic chains have recently been studied in other models as well. An illustrative example could be a systematic study of the spectroscopic properties of the nuclei from Pt down to Yb isotopes reported in Ref. [26], where the same technique as in the former paper was applied to describe the rotational bands of these nuclei, with the microscopic input being the Gogny effective interaction [28]. Moreover, more recently an analysis based on a phenomenological Bohr Hamiltonian approach using a chosen collective potential has been presented [27] to study the spectroscopy of Hf nuclei.

In this study, coherent state formalism is used to obtain experimental observables of real nuclei. Hf isotopes are selected since the rotational character is dominant for these isotopes. The rest of the paper as follows. In Section 2, moment of inertias and energy of the levels as a function of deformation parameter are found for ground-state and excited states by using the IBM Hamiltonian written for the $SU(3)$–$O(6)$ transitional region. Intrinsic quadrupole moments are obtained in a closed form in Section 3. Finally the last section is devoted to application to the Hf isotopes and to discussion of the results.

2. Coherent state formalism

In the $SU(3)$–$O(6)$ region, the IBM-1 Hamiltonian in the multipole form [4] is given by

$$\hat{H} = \kappa_3 \hat{Q} \cdot \hat{Q} - \kappa' \hat{L} \cdot \hat{L} + \kappa'' P^\dagger \cdot \tilde{P} \tag{1}$$

where quadrupole ($\hat{Q}$), angular momentum ($\hat{L}$) and pairing ($\hat{P}$) operators are defined as