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Confinement–deconfinement phase transition in hot and dense QCD at large N

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Abstract

We conjecture that the confinement–deconfinement phase transition in QCD at large number of colors N and $N_f \ll N$ at $T \neq 0$ and $\mu \neq 0$ is triggered by the drastic change in θ behavior. The conjecture is motivated by the holographic model of QCD where confinement–deconfinement phase transition indeed happens precisely at the value of temperature $T = T_c$ where θ dependence experiences a sudden change in behavior [A. Parnachev, A. Zhitnitsky, arXiv: 0806.1736 [hep-ph]]. The conjecture is also supported by quantum field theory arguments when the instanton calculations (which trigger the θ dependence) are under complete theoretical control for $T > T_c$, suddenly break down immediately below $T < T_c$ with sharp changes in the θ dependence. Finally, the conjecture is supported by a number of numerical lattice results. We employ this conjecture to study confinement–deconfinement phase transition of dense QCD at large μ in large N limit by analyzing the θ dependence. We find that the confinement–deconfinement phase transition at $N_f \ll N$ happens at very large quark chemical potential $\mu_c \sim \sqrt{N}\Lambda_{\rm QCD}$. This result agrees with recent findings by McLerran and Pisarski [L. McLerran, R.D. Pisarski, Nucl. Phys. A 796 (2007) 83]. We also speculate on case when $N_f \sim N$.

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1. Introduction

Understanding the phase diagram at nonzero external parameters T, μ is one of the most difficult problem in QCD. Obviously, this area is a prerogative of numerical lattice computations.

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However, some insights about the basic features of the phase diagram may be inferred by using some analytical approaches. In particular, some qualitative questions can be formulated and answered by considering a theory with large number of colors N or/and flavors N_f, see recent papers on then subject [1–3] and references on previous works therein. Generically, to study a phase transition one should find an appropriate order parameter. It is easy to find an order parameter for gluodynamics when light quarks are not present in the system. If massless quarks are introduced into the system, one can study a chiral phase transition and use the chiral condensate as an order parameter. For massive, but light quarks this is not an option. However, in the limit of very large N one can consider the free energy as an order parameter. In confined phase it is order of one, while in deconfined phase it is order of $\sim N^2$. Small number of flavors $N_f \ll N$ (massless or massive quarks) does not change the basic picture.

We formulate a different criteria for confinement–deconfinement phase transition, and therefore we use a different order parameter to analyze the phase transition. The new criteria is based on observation that the deconfined phase transition is always accompanied by very sharp changes in θ behavior which represents our basic conjecture. Therefore, in principle, if our conjecture is correct, one can use any order parameter which nontrivially depends on θ and study this dependence on two sides of the phase transition line. Very natural question immediately comes into mind: why and how these two different things (phase transition vs sharp θ changes) could be linked? What is the basic motivation for this proposal? First of all, this criteria is motivated by the observation that in holographic model of QCD the confinement–deconfinement phase transition happens precisely at the value of temperature $T = T_c$, where θ dependence experiences a sudden change in behavior [1]. Secondly, the proposal is supported by the numerical lattice results [4–8], see also a review article [9], which unambiguously suggest that the topological fluctuations are strongly suppressed in deconfined phase, and this suppression becomes more severe with increasing N. These general features observed in the lattice simulations have very simple explanation within our proposal on the origin of the confinement–deconfinement phase transition, see next section for details. Finally, our new criteria is based on a physical picture which can be shortly summarized as follows.

For sufficiently high temperatures $T > T_c$ the instanton gas is dilute with density $\sim e^{-\gamma(T)N}$ which implies a strong suppression[1] of the topological fluctuations at large N where $\gamma(T) > 0$, see below for details on structure of $\gamma(T)$-function. The calculations in this region are under complete field theoretical control and the vacuum energy has a nice analytic behavior $\sim \cos\theta\, e^{-\gamma(T)N}$ as function of θ. At the critical value of temperature, $T = T_c$ where $\gamma(T)$ changes the sign, the instanton expansion breaks down and one should naturally expect that at $T = T_c$ there should be a sharp transition in θ behavior as simple formula $\sim \cos\theta$ can only be valid when the instanton gas is dilute and semiclassical calculations are justified which is obviously not the case for $T < T_c$. Therefore, it is naturally to associate sharp changes in θ behavior with confinement–deconfinement transition, just as in the holographic model [1]. There is a very narrow window of temperatures in deconfined phase, $0 < (T - T_c)/T_c \leqslant 1/N$, when the instanton expansion is not valid. This vicinity of T_c is extremely interesting, see our comments about physics in this region in conclusion. This region shrinks to a point at $N = \infty$.

The main goal of this paper is to apply this criteria to the region with large chemical potential at large N and $N_f \ll N$ and make a specific prediction on magnitude $\mu_c(T)$ for confinement–deconfinement transition line at large μ and sufficiently small $T \ll \mu$. The corresponding esti-

[1] See [10] and references therein for earlier discussions on the subject.