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Invariant approach to $\mathcal{CP}$ in unbroken $\Delta(27)$

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Abstract

The invariant approach is a powerful method for studying $\mathcal{CP}$ violation for specific Lagrangians. The method is particularly useful for dealing with discrete family symmetries. We focus on the $\mathcal{CP}$ properties of unbroken $\Delta(27)$ invariant Lagrangians with Yukawa-like terms, which proves to be a rich framework, with distinct aspects of $\mathcal{CP}$, making it an ideal group to investigate with the invariant approach. We classify Lagrangians depending on the number of fields transforming as irreducible triplet representations of $\Delta(27)$. For each case, we construct $\mathcal{CP}$-odd weak basis invariants and use them to discuss the respective $\mathcal{CP}$ properties. We find that $\mathcal{CP}$ violation is sensitive to the number and type of $\Delta(27)$ representations.

1. Introduction

The origin and nature of $\mathcal{CP}$ and its violation remain a mystery both within and beyond the Standard Model (SM). In addressing the question of $\mathcal{CP}$ it was observed some time ago that phases which appear in the Yukawa matrices for example are not robust indicators of $\mathcal{CP}$ violation since their appearance is dependent on the choice of basis. On the other hand, physical $\mathcal{CP}$ violating observables only depend on particular combinations of Yukawa matrices which are in-

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variant under different choices of basis. Such weak basis invariants, which have the property that they are zero if $\mathcal{CP}$ is conserved and non-zero if $\mathcal{CP}$ is violated therefore provide unambiguous signals of $\mathcal{CP}$ violation which are closely related to experimentally measurable quantities. The use of such $\mathcal{CP}$-odd weak-basis invariants (CPIs), rather than particular phases in a given basis, is generally referred to as the Invariant Approach (IA) to $\mathcal{CP}$ violation.

In the IA to $\mathcal{CP}$ violation [1], one starts by separating the full Lagrangian of the theory in two parts, one denoted $\mathcal{L}_{\mathcal{CP}}$ that is known to conserve $\mathcal{CP}$, typically the kinetic terms and pure gauge interactions [2],[1] and the remaining Lagrangian, denoted $\mathcal{L}_{rem.}$. The crucial point is that $\mathcal{L}_{\mathcal{CP}}$ allows for many different $\mathcal{CP}$ transformations and as a result, $\mathcal{CP}$ is violated if and only if none of these $\mathcal{CP}$ transformations leaves $\mathcal{L}_{rem.}$ invariant. In the case of the SM, $\mathcal{L}_{\mathcal{CP}}$ includes the gauge interactions and the kinetic energy terms, while the relevant components of $\mathcal{L}_{rem.}$ are the Yukawa interactions. Using the IA, one can readily derive [1] some specific conditions that the Yukawa couplings have to satisfy in order to have $\mathcal{CP}$ invariance. It is well known that the Yukawa couplings in the SM have a large redundancy which results from the freedom that one has to make redefinitions of the fermion fields which leave the gauge interactions invariant but change e.g. the quark Yukawa couplings Y_u, Y_d without changing the Physics. The great advantage of the IA is that it allows one to derive CPIs which, if non-vanishing, imply $\mathcal{CP}$ violation. In the SM, it has been shown [1] that the relevant CPI is $\mathrm{Tr}\,[H_u, H_d]^3$, where we define the Hermitian combinations $H_u \equiv Y_u Y_u^\dagger$ and $H_d \equiv Y_d Y_d^\dagger$. For the 3 fermion generation case this CPI leads to the Jarlskog invariant [4]. The IA can be applied to any extension of the SM, in particular to extensions of the SM with Majorana neutrinos [5].

It should be emphasised that the IA not only enables one to verify whether a given Lagrangian violates $\mathcal{CP}$, but also provides an idea of how suppressed $\mathcal{CP}$ violation might be. A notable example is the possibility of showing why $\mathcal{CP}$ in the SM is too small to generate the baryon asymmetry of the Universe (BAU). One simply observes that the dimensionless number $\mathrm{Tr}\,[M_u M_u^\dagger, M_d M_d^\dagger]^3/v^{12}$ is of order 10^{-20}, where we used the Hermitian quark mass matrices and $v = 246$ GeV denotes the scale of electroweak symmetry breaking. This dimensionless number should be compared to the size of observed BAU, $n_B/n_\gamma \simeq 10^{-10}$ [6]. The IA, leading to basis invariant quantities, also identifies what combination of parameters are physical such that, e.g. there is no need to count how many phases can be eliminated through rephasing, which can be laborious in complicated Lagrangian, and specially in the presence of family symmetries.

Recently [7] the use of CPIs, valid for any choice of $\mathcal{CP}$ transformation, was advocated as a powerful approach to studying specific models of $\mathcal{CP}$ violation in the presence of discrete family symmetries. Examples based on A_4 and $\Delta(27)$ family symmetries were discussed and it was shown how to obtain several known results in the literature. In addition, the IA was used to identify how explicit (rather than spontaneous) $\mathcal{CP}$ violation arises, which is geometrical in nature, i.e. persisting for arbitrary couplings in the Lagrangian.

Here we intend both to further highlight the usefulness of the IA in dealing with discrete family symmetries and also to systematically explore the $\mathcal{CP}$ properties of $\Delta(27)$. By using the IA, we are able to construct CPIs independently of the specific group and need to consider the group details only to compute coupling matrices by using the respective Clebsch–Gordan coefficients in any particular basis. By combining the coupling matrices with the CPIs, basis-independent quantities are obtained which indicate if there is $\mathcal{CP}$ violation.

[1] The use of $\mathcal{CP}$-like transformations that include a family symmetry transformation was introduced in [3], in an attempt to obtain a connection between quark masses and mixing angles.